MINISTRY OF EDUCATION AND TRAINING

VIETNAM ACADEMY OF SCIENCE AND TECHNOLOGY

GRADUATE UNIVERSITY OF SCIENCE AND TECHNOLOGY

PHAM NGOC CHUNG

INVESTIGATION OF TEMPERATURE RESPONSES OF SMALL SATELLITES IN LOW EARTH ORBIT SUBJECTED TO THERMAL LOADINGS FROM SPACE ENVIRONMENT

Major: Engineering Mechanics

Code: 9 52 01 01

SUMMARY OF THE DOCTORAL THESIS

Hanoi – 2019

The thesis has been completed at Graduate University of Science and Technology, Vietnam Academy of Science and Technology

Supervisor 1: Prof.Dr.Sc. Nguyen Dong Anh
Supervisor 2: Assoc.Prof.Dr. Dinh Van Manh

Reviewer 1: Prof.Dr. Tran Ich Thinh
Reviewer 2: Prof.Dr. Nguyen Thai Chung
Reviewer 3: Assoc.Prof.Dr. Dao Nhu Mai

The thesis is defended to the thesis committee for the Doctoral Degree, at Graduate University of Science and Technology - Vietnam Academy of Science and Technology, on Date.....Month.....Year 2019

Hardcopy of the thesis can be found at:

- Library of Graduate University of Science and Technology
- National Library of Vietnam

INTRODUCTION

1. The rationale for the thesis

In the past decades, the problem of nonlinear behavior analysis of dynamical systems is of interest of researchers from over the world. In the field of space technology, satellite thermal analysis is one of the most complex but important tasks because it involves the operation of satellite equipment in orbit. To explore the thermal behavior of a satellite, one can use numerical computation tools packed in a specialized software. The numerical computation-based approach, however, needs a lot of resources of computer. When changing system parameters, the calculation process of thermal responses may require a new iteration corresponding to the parameter data under consideration. This leads to an "expensive" cost of computation time. Another approach based on analytical methods can take advantage of the convenience and computation time, because it can quickly estimate thermal responses of a certain satellite component with a desired accuracy. Until now, there are very little effective analytical tools to solve the problem of satellite thermal analysis because of the presence of quartic nonlinear terms related to heat radiation. For the above reasons, I have chosen a subject for my thesis, entitled "*Investigation of temperature responses of small satellites in Low Earth Orbit subjected to thermal loadings from space environment*" by proposing an efficient analytical tool, namely, a dual criterion equivalent linearization method which is developed recently for nonlinear dynamical systems.

2. The objective of the thesis

- Establishing thermal models of single-node, two-node and many-node associated with different thermal loading models acting on a small satellite in Low Earth Orbit.

- Finding analytical solutions of equations of thermal balance for small satellites by the dual criterion equivalent linearization method.

- Exploring quantitative and qualitative behaviors of satellite temperature in the considered thermal models.

3. The scope of the thesis

The thesis is focused to investigate characteristics of thermal responses of small satellites in Low Earth Orbit; the investigation scope includes single-node, two-node, six-node and eight-node models.

3. The research methods in the thesis

The thesis uses analytical methods associated with numerical methods:

- The method of equivalent linearization; Grande's approximation methods;

- The 4^{th} order Runge-Kutta method for solving differential equations of thermal balance.

- The Newton-Raphson method for solving nonlinear algebraic systems obtained from linearization processes of thermal balance equations.

4. The outline of the thesis

The thesis is divided into the following parts: Introduction; Chapters 1, 2, 3 and 4; Conclusion; List of research works of author related to thesis contents; and References.

CHAPTER 1. AN OVERVIEW OF SATELLITE THERMAL ANALYSIS PROBLEMS

- Chapter 1 presents an overview of the thermal analysis problem for small satellites in Low Earth Orbit.

- In Low Earth Orbit, a satellite is experienced three main thermal loadings from space environment, namely, solar irradiation, Earth's albedo and infrared radiation. In the thesis, these loadings are formulated in the form of analytical expressions, and they can be easily processed in both analytical and numerical analysis.

- The author presents the thermal modeling process for small satellites based upon the lumped parameter method to obtain nonlinear differential equations of thermal balance of nodes. The author has introduced physical expressions of thermal nodes in detail, for example heat capacity, conductive coupling coefficient, radiative coupling coefficient. For satellites in Low Earth Orbit, the main mechanisms of heat transfer are thermal radiation and conduction through material medium of spacecraft (here, convection is considered negligible).

CHAPTER 2. ANAYSIS OF THERMAL RESPONSE OF SMALL SATELLITES USING SINGLE-NODE MODEL

2.1. Problem

Thermal analysis is one of the important tasks in the process of thermal design for satellites because it involves the temperature limit and stable operation of satellite equipment. For small satellites, the satellite can be divided into several nodes in the thermal model. In this chapter, a single-node model is considered. The meaning of single-node model is as follows: (i) this is a simple model that allows estimating temperature values of a satellite, a certain component or device; (ii) the model supports to reduce the "cost" of computation in the pre-design phase of the satellite, especially, temperature estimation with assumed heat inputs in thermodynamic laboratories.

For single-node model, a satellite is considered as a single body that can exchange radiation heat in the space environment. According to the second law of thermodynamics, we obtain an equation of energy balance for the satellite with a single-node model as follows:

$$C\dot{T} = -A_{sc}\varepsilon\sigma T^4 + Q_s f_s(\nu t) + Q_a f_a(\nu t) + Q_e, \tag{2.1}$$

where C is the heat capacity, $T = T(t)$ is nodal temperature, the notation A_{sc} denotes the surface area of the node in the model, ε is the emissivity, $\sigma = 5.67\times10^{-8}$ WK^{-4}m^{-2} is the Stefan–Boltzmann constant; the quantity $Q_s f_s(\nu t) + Q_a f_a(\nu t) + Q_e$ represents a sum of external thermal loads, includes solar irradiation $Q_s f_s(\nu t)$, Earth's albedo $Q_a f_a(\nu t)$ and Earth's infrared radiation Q_e.

2.2. External thermal loadings

- Solar irradiation: When the satellite is illuminated, the solar irradiation thermal loading $Q_s f_s(\nu t)$ differs from zero. Against, this loading will vanish as the satellite is in the fraction of orbit in eclipse, it means:

$$Q_{sol} = Q_s f_s(\nu t) = G_s A_{sp}\alpha_s f_s(\nu t), \tag{2.2}$$

where G_s is the mean solar irradiation and A_{sp} is the satellite surface projected in the Sun's direction; $f_s(\nu t)$ represents the day-to-night variations of the solar irradiation, this function $f_s(\nu t)$ has a square wave shape, $f_s(\nu t) = 1$ for $0 \le \nu t \le \mu\pi$ and $(1-\mu/2)2\pi \le \nu t \le 2\pi$, $f_s(\nu t) = 0$ for $\mu\pi < \nu t < (1-\mu/2)2\pi$, in an orbital period. $\mu = P_{il}/P_{orb}$ is the ratio of the illumination period P_{il} (s) to the orbital period P_{orb} (s).

- Earth's albedo radiation: When the Sun illuminates the Earth, a part of solar energy is absorbed by the Earth's surface, the remaining part is reflected into space. The reflection will affect directly on the satellite, known as the Earth's albedo radiation. The albedo loading acting on the satellite is expressed as follows:

$$Q_{alb} = Q_a f_a(\nu t) = a_e G_s A_{sc} F_{se} \alpha_s f_a(\nu t), \tag{2.3}$$

in which a_e is albedo factor; A_{sc} represents the surface area of the node; F_{se} is the view factor from the whole satellite to the Earth; $f_a(\nu t)$ denotes the day-to-night variations of the albedo thermal loads, $f_a(\nu t) = \cos(\nu t)$ for $0 \le \nu t \le \pi/2$ and $3\pi/2 \le \nu t \le 2\pi$, $f_a(\nu t) = 0$ for $\pi/2 < \nu t < 3\pi/2$.

- Infrared radiation: The Earth's infrared radiation Q_e can be evaluated as

$$Q_e = \varepsilon A_{sc} F_{se} \sigma T_e^4, \tag{2.4}$$

where T_e is the Earth's equivalent black-body temperature.

We introduce the following dimensionless quantities:

$$\tau = \nu t,\ \theta = T(t)/\beta,\ \gamma_1 = Q_s/\nu\beta C,\ \gamma_2 = Q_a/\nu\beta C,\ \gamma_3 = Q_e/\nu\beta C \tag{2.5}$$

where

$$\nu = 2\pi/P_{orb},\ \ \beta = (\nu C/A_{sc}\varepsilon\sigma)^{1/3}. \tag{2.6}$$

Using (2.5), the equation of thermal balance (2.1) is transformed in the following dimensionless form

$$\frac{d\theta}{d\tau} = -\theta^4 + \gamma_1 f_s(\tau) + \gamma_2 f_a(\tau) + \gamma_3. \tag{2.7}$$

In this chapter, the author proposes a new approach to find approximate periodic solutions of Eq. (2.7) using the dual criterion of equivalent linearization method studied recently for random nonlinear vibrations. The main idea of this approach is based on the replacement of origin nonlinear system under external loadings that can be deterministic or random functions by a linear one under the same excitation for which the coefficients of linearization can be found from proposed dual criterion for satellite thermal analysis.

2.3. The dual criterion of equivalent linearization

We consider the first order differential equation of the form

$$\frac{d\theta}{d\tau} + f\left(\theta\right) = \xi\left(\tau\right), \tag{2.8}$$

where $f\left(\theta\right)$ is a nonlinear function of the argument θ and $\xi\left(\tau\right)$ is an external loading that can be deterministic or random functions. The original Eq. (2.8) is linearized to become a linear equation of the following form

$$\frac{d\theta}{d\tau} + a\theta + b = \xi\left(\tau\right), \tag{2.9}$$

where two equivalent linearization coefficients a, b are found from a specified criterion.

In the linearization process of the thesis, the dual criterion has obtained from two steps of replacement as follows:

- *The first step*: the nonlinear function $f\left(\theta\right)$ representing the thermal radiation term is replaced by a linear one $a\theta + b$, in which $a,\ b$ are the linearization coefficients.

- *The second step:* The linear function $a\theta + b$ is replaced by another nonlinear one of the form $\lambda f\left(\theta\right)$ that can be considered as a function belonging to the same class of the original function $f\left(\theta\right)$, with the scaling factor λ, in which the linearization coefficients $a,\ b$ and λ are found from the following compact criterion,

$$J = \left(1-\rho\right)\left\langle \left(f\left(\theta\right) - a\theta - b\right)^2 \right\rangle + \rho\left\langle \left(a\theta + b - \lambda f\left(\theta\right)\right)^2 \right\rangle \to \min_{a,b,\lambda}, \tag{2.10}$$

where the parameter ρ takes two values, 0 or 1/2. It is seen from Eq. (2.10) that when $\rho = 0$, we obtain the conventional mean-square error criterion of equivalent linearization. When $\rho = 1/2$, we obtain the dual criterion proposed in work by Anh et al. in 2012. The criterion (2.10) contains both conventional and dual criteria of equivalent linearization in a compact form.

The criterion (2.10) leads to the following system for determining unknowns a, b and λ

$$\frac{\partial J}{\partial a} = 0, \quad \frac{\partial J}{\partial b} = 0, \quad \frac{\partial J}{\partial \lambda} = 0. \tag{2.11}$$

Equation (2.11) gives the result of linearization coefficient a, b,

$$a = \frac{1-\rho}{1-\rho\Phi} \frac{\langle \theta f(\theta)\rangle - \langle \theta\rangle \langle f(\theta)\rangle}{\langle \theta^2\rangle - \langle \theta\rangle^2}, \quad b = \frac{1-\rho}{1-\rho\Phi} \frac{\langle \theta^2\rangle \langle f(\theta)\rangle - \langle \theta\rangle \langle \theta f(\theta)\rangle}{\langle \theta^2\rangle - \langle \theta\rangle^2} \tag{2.12}$$

and, the return coefficient λ

$$\lambda = \frac{1-\rho}{1-\rho\Phi} \left[\frac{\langle \theta f(\theta)\rangle}{\langle f^2(\theta)\rangle} \frac{\left(\langle \theta f(\theta)\rangle - \langle \theta\rangle \langle f(\theta)\rangle\right)}{\langle \theta^2\rangle - \langle \theta\rangle^2} + \frac{\langle f(\theta)\rangle}{\langle f^2(\theta)\rangle} \frac{\left(\langle \theta^2\rangle \langle f(\theta)\rangle - \langle \theta\rangle \langle \theta f(\theta)\rangle\right)}{\langle \theta^2\rangle - \langle \theta\rangle^2} \right] \tag{2.13}$$

where it is denoted,

$$\Phi = \frac{\left(\langle \theta\rangle \langle f(\theta)\rangle - \langle \theta f(\theta)\rangle\right)^2}{\left(\langle \theta^2\rangle - \langle \theta\rangle^2\right) \langle f^2(\theta)\rangle} + \frac{\langle f(\theta)\rangle^2}{\langle f^2(\theta)\rangle}. \tag{2.14}$$

In the framework of the thermal balance equation (2.7), the function $f(\theta)$ is taken to be $f(\theta) = \theta^4$. In next subsection, we will find approximate responses of Eq. (2.7) using the generalized results (2.12-2.14).

2.4. An approximate solution for the thermal balance equation

It is seen that, due to the periodicity of two input functions $f_s(\tau)$, $f_a(\tau)$ determined from Eqs. (2.2) and (2.3), they can be expressed as Fourier expansions

$$f_s(\tau) = \mu + \frac{2}{\pi}\sin\mu\pi\cos\tau + \sum_{k=2}^{\infty}\frac{2}{k\pi}\sin k\mu\pi\cos k\tau, \qquad (2.15)$$

$$f_a(\tau) = \frac{1}{\pi} + \frac{1}{2}\cos\tau - \sum_{k=1}^{\infty}\frac{2}{\pi(4k^2-1)}\cos(2k\tau + k\pi). \qquad (2.16)$$

The terms of two series tend to zero as the index k increases. Thus, for simplicity, in the later calculation, only the first harmonic terms of each series will be retained. Hence, Eq. (2.7) can be rewritten as

$$\frac{d\theta}{d\tau} = -\theta^4 + P + H\cos\tau, \qquad (2.17)$$

where it is denoted

$$P = \mu\gamma_1 + \frac{1}{\pi}\gamma_2 + \gamma_3,\ H = \frac{2}{\pi}\gamma_1\sin\mu\pi + \frac{1}{2}\gamma_2. \qquad (2.18)$$

The solution of Eqs. (2.9), with $\xi(\tau) = P + H\cos\tau$, is expressed as

$$\theta(\tau) = R + A\cos\tau + B\sin\tau, \qquad (2.19)$$

where R, A, B are determined by substituting Eqs. (2.19) (with $\xi(\tau) = P + H\cos\tau$) into Eq. (2.9) and equating coefficients of corresponding harmonic terms

$$R = \frac{P-b}{a},\ A = \frac{a}{1+a^2}H,\ B = \frac{1}{1+a^2}H. \qquad (2.20)$$

Substituting expression $f(\theta) = \theta^4$ into Eqs. (2.12-2.14), after some calculations involving the average response, we obtain the nonlinear algebraic system for the linearization coefficients a and b as follows:

$$a=\frac{1-\rho}{1-\rho\Phi}\frac{P-b}{a}\left[4\left(\frac{P-b}{a}\right)^2+\frac{3H^2}{1+a^2}\right],\ b=\frac{1-\rho}{1-\rho\Phi}\left[-3\left(\frac{P-b}{a}\right)^4+\frac{3}{8}\frac{H^4}{\left(1+a^2\right)^2}\right],\tag{2.21}$$

where

$$\Phi=\frac{R^8+14R^6\left(A^2+B^2\right)+\frac{87}{4}R^4\left(A^2+B^2\right)^2+\frac{27}{4}R^2\left(A^2+B^2\right)^3+\frac{9}{64}\left(A^2+B^2\right)^4}{R^8+14R^6\left(A^2+B^2\right)+\frac{105}{4}R^4\left(A^2+B^2\right)^2+\frac{35}{4}R^2\left(A^2+B^2\right)^3+\frac{35}{128}\left(A^2+B^2\right)^4}.\tag{2.22}$$

Because system (2.21) is a nonlinear algebraic equations system for linearization coefficients $a,\ b$ in the closed form, this system can be solved by the Newton–Raphson iteration method. Then using (2.20), we obtain the approximate solution (2.19) of the system (2.7). It is noted again that the conventional and dual linearization coefficients are obtained from Eq. (2.21) by setting $\rho=0$ and 1/2, respectively.

Solution obtained from Grande's approach in steady-state regime is

$$\eta_s\left(\tau\right)=\frac{H}{1+16\bar{\theta}^6}\left(4\bar{\theta}^3\cos\tau+\sin\tau\right).\tag{2.23}$$

The temperature fluctuation amplitudes χ_G of $\theta\left(\tau\right)$ received from Grande's approach (2.23) and χ_{DC} derived from the solution (2.21) of the compact dual criterion (2.10) are, respectively,

$$\chi_G=\frac{H}{\sqrt{1+16\bar{\theta}^6}},\qquad \chi_{DC}=\frac{H}{\sqrt{1+a^2}}.\tag{2.24-2.25}$$

In the next section, we compare results of thermal response $\theta\left(\tau\right)$ obtained by the dual linearization, conventional linearization, and Grande’s approach with the numerical solution of the Runge–Kutta method.

2.5. Thermal analysis for small satellites with single-node model

The results in Figures 2.1 and 2.2 exhibit that the graphs of temperature obtained from the method of equivalent linearization and

Grande's approach are quite close to the one obtained from the Runge–Kutta method. Taking reference of the thermal response obtained by the Runge-Kutta method, the dual criterion of equivalent linearization gives smaller errors than other methods when the nonlinearity of the system increases, namely, when the heat capacity C varies in the range $[1.0, 3.0] \times 10^4$ (JK^{-1}).

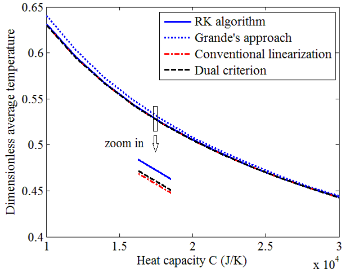

Figure 2.1. Dimensionless average temperature with various methods.

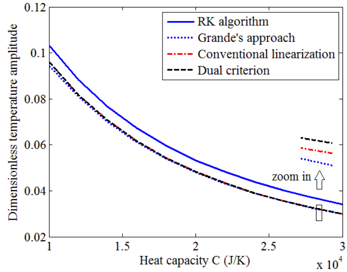

Figure 2.2. Dimensionless temperature amplitude with various methods.

Table 2.1. Dimensionless average temperature θ with various values of the heat capacity C

C	$\langle\theta\rangle_{RK}$	$\langle\theta\rangle_{G}$	**Error (%)**	$\langle\theta\rangle_{CL}$	**Error (%)**	$\langle\theta\rangle_{DC}$	**Error (%)**
10000	0.6313	0.640598492	1.4702	0.629860124	0.2307	0.630153556	0.1842
12000	0.5957	0.602826261	1.1923	0.594551363	0.1967	0.594743420	0.1645
14000	0.5671	0.572633257	0.9714	0.566148522	0.1720	0.566276351	0.1495
16000	0.5434	0.547704006	0.7988	0.542538575	0.1519	0.542623672	0.1362
18000	0.5231	0.526617245	0.6640	0.522439261	0.1347	0.522499387	0.1232
20000	0.5056	0.508443360	0.5581	0.505016142	0.1197	0.505058661	0.1113
22000	0.4902	0.492543983	0.4742	0.489696381	0.1066	0.489727041	0.1004
24000	0.4765	0.478463520	0.4071	0.476069908	0.0953	0.476092423	0.0905
26000	0.4642	0.465866479	0.3526	0.463833290	0.0853	0.463850105	0.0817
28000	0.4531	0.454499317	0.3081	0.452755823	0.0767	0.452768580	0.0739
30000	0.4430	0.444166187	0.2712	0.442658334	0.0692	0.442668151	0.0669

Table 2.1 reveals that, in the considered range of the heat capacity C, the maximal errors of the dual and conventional linearization criteria are about 0.1842% and 0.2307%, respectively, whereas the maximal error of the Grande's approach is about 1.4702%.

2.6. Conclusions of Chapter 2

This chapter is devoted to the use of the new method of equivalent linearization in finding approximate solutions of small satellite thermal problems in the Low Earth Orbit. A compact dual criterion of equivalent linearization is developed to contain both the convention and dual criteria for single-node model. A system of algebraic equations for linearization coefficients is obtained in the closed form and can be then solved by an iteration method. Numerical simulation results show the reliability of the linearization method. The graphs of temperature obtained from the method of equivalent linearization and Grande's approach are quite close to the one obtained from the Runge–Kutta method. In addition, the dual criterion yields smaller errors than those when the nonlinearity of the system increases, namely, when the heat capacity C varies in the range $[1.0, 3.0] \times 10^4$ JK^{-1}).

The results of Chapter 2 are published in two papers [1] and [7] in the *List of published works related to the author's thesis*.

CHAPTER 3. ANALYSIS OF THERMAL RESPONSE OF SMALL SATELLITES USING TWO-NODE MODEL

3.1. Problem

For purpose of well-understanding on temperature behaviors of the satellite, many-node models may be proposed and studied in different satellite missions.

In this chapter, the author studies a two-node model for small spinning satellites. The satellite is modeled as an isothermal body with two nodes, namely, outer and inner nodes. The outer node, representing the shell, the solar panels and any external device located on the outer surface of the satellite, and

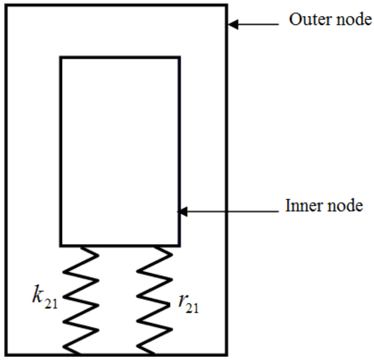

Figure 3.1. Two-node system model

the inner node which includes all equipment within it (for example, payload and electronic devices). The thermal interaction between two nodes can be modeled as a two-degree-of-freedom system in which the link between them can be considered as linear elastic link for conduction phenomena and nonlinear elastic link for radiation phenomena, as illustrated in Figure 3.1.

Let C_1 and C_2 be the thermal capacities of the outer and the inner nodes, respectively, and T_1 and T_2 their temperatures. The equation of the energy balance for the two-node model takes the following form

$$\begin{aligned} C_1\dot{T}_1 &= k_{21}\left(T_2 - T_1\right) + r_{21}\left(T_2^4 - T_1^4\right) - A_{sc}\varepsilon\sigma T_1^4 + Q_s\, f_s\left(\nu t\right) + Q_a\, f_a\left(\nu t\right) + Q_e, \\ C_2\dot{T}_2 &= -k_{21}\left(T_2 - T_1\right) - r_{21}\left(T_2^4 - T_1^4\right) + Q_{d2}, \end{aligned} \tag{3.1}$$

where $Q_s f_s\left(\nu t\right)$, $Q_a f_a\left(\nu t\right)$, Q_e is the solar irradiation, albedo and Earth's infrared radiation, respectively; and, Q_{d2} is the internal heat dissipation which is assumed to be undergone a constant heat dissipation level.

The equation of thermal balance (3.1) can be transformed in the following dimensionless form

$$
\begin{aligned}
c\frac{d\theta_1}{d\tau} &= k\left(\theta_2-\theta_1\right)+r\left(\theta_2^4-\theta_1^4\right)-\theta_1^4+\gamma_1 f_s\left(\tau\right)+\gamma_2 f_a\left(\tau\right)+\gamma_3, \\
\frac{d\theta_2}{d\tau} &= -k\left(\theta_2-\theta_1\right)-r\left(\theta_2^4-\theta_1^4\right)+\gamma_4,
\end{aligned} \tag{3.2}
$$

where $\theta_1=\theta_1\left(\tau\right)$, $\theta_2=\theta_2\left(\tau\right)$ are dimensionless temperature functions of the dimensionless time τ; and it is denoted

$$
\begin{aligned}
&\theta_1=T_1\left(t\right)/\beta,\ \theta_2=T_2\left(t\right)/\beta,\ \beta=\left[C_2\nu/\left(A_{sc}\varepsilon\sigma\right)\right]^{1/3},\ \tau=\nu t, \\
&\nu=2\pi/P_{orb},\ c=C_1/C_2,\ k=k_{21}/\nu C_2,\ r=r_{21}\beta^3/\nu C_2, \\
&\gamma_1=Q_s/\left(\beta C_2\nu\right), \gamma_2=Q_a/\left(\beta C_2\nu\right), \gamma_3=Q_p/\left(\beta C_2\nu\right), \\
&\gamma_4=Q_{d2}/\left(\beta C_2\nu\right).
\end{aligned} \tag{3.3}
$$

The author will extend the dual criterion developed in Chapter 2 for the two-node model (3.2), to find approximation of the satellite thermal system.

3.2. Extension of dual equivalent linearization for two-node model

For the equivalent linearization approach, to simplify the process of linearization, a preprocessing step in nonlinear terms of the original system is carried out to get an equivalent system in which each differential equation contains only one nonlinear term. On the basic of the dual criterion, as presented in Chapter 2 [see (2.10)], a closed form of linearization coefficients system is obtained and solved by a Newton–Raphson iteration procedure.

After finding the linearization coefficients, we obtain the approximate thermal response of nodes [2].

3.3. Thermal analysis for small satellites with two-node model

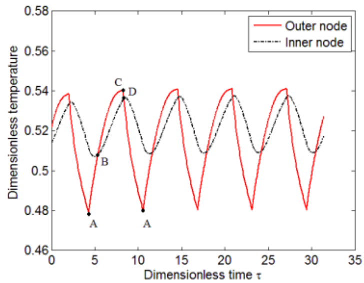

Figure 3.2. Inner and outer nodes' dimensionless temperatures as functions of dimensionless time

In Fig. 2, temperature calculations are performed for the nonlinear system (3.2) using the Runge–Kutta algorithm corresponding to 5 orbital periods. Several characteristic points such as A, B, C and D of the satellite's orbit are remarked. The letter A shows the sunrise point whereas the letter C is the sunset point in the orbit. Two letters B and D are intersection points of two outer and inner temperature curves in time.

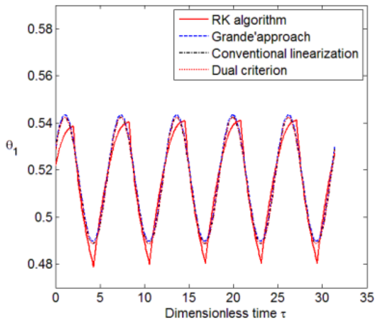

Figure 3.3. Dimensionless temperature evolution of $\theta_1(\tau)$ by various methods

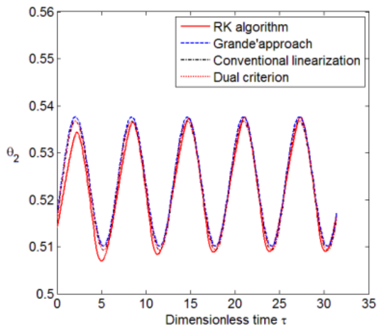

Figure 3.4. Dimensionless temperature evolution of $\theta_2(\tau)$ by various methods

To evaluate the efficiency of the equivalent linearization method, we show the computation time (solution time) for various methods as shown in Figure 3.5. For reference solution time of the dual method, it is seen that the computation time of the RK algorithm is quite large in comparison with those of remaining methods.

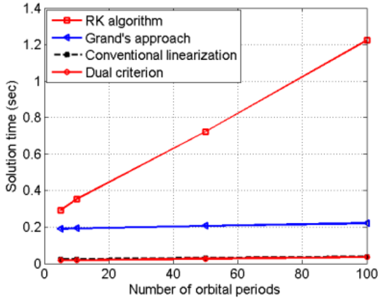

Figure 3.5. Comparison of solution time of various methods via the number of orbital periods.

Table 3.1. Outer node's dimensionless average temperature with various values of thermal capacity C_2 ($\langle\theta\rangle_{RK}$: Runge–Kutta method; $\langle\theta\rangle_{G}$: Grande's approach; $\langle\theta\rangle_{CL}$: Conventional linearization; $\langle\theta\rangle_{DC}$: Dual criterion method).

C	$\langle\theta\rangle_{RK}$	$\langle\theta\rangle_{G}$	**Error (%)**	$\langle\theta\rangle_{CL}$	**Error (%)**	$\langle\theta\rangle_{DC}$	**Error (%)**
10000	0.6443	0.650838235	1.0079	0.649431902	0.7896	0.649437815	0.7906
12000	0.6062	0.612462262	1.0307	0.611174293	0.8183	0.611179566	0.8191
14000	0.5757	0.581786667	1.0558	0.580583551	0.8469	0.580588397	0.8477
16000	0.5505	0.556458963	1.0810	0.555319285	0.8740	0.555323833	0.8748
18000	0.5291	0.535035168	1.1050	0.533944706	0.8989	0.533949036	0.8997
20000	0.5108	0.516570807	1.1274	0.515519787	0.9216	0.515523953	0.9225
22000	0.4947	0.500417309	1.1480	0.499398797	0.9422	0.499402835	0.9430
24000	0.4805	0.486111797	1.1669	0.485120729	0.9607	0.485124665	0.9615
26000	0.4677	0.473313417	1.1841	0.472345996	0.9773	0.472349847	0.9782
28000	0.4562	0.461764572	1.1999	0.460817880	0.9924	0.460821660	0.9932
30000	0.4458	0.451266287	1.2143	0.450338031	1.0061	0.450341749	1.0069

Calculation data corresponding to the characteristics of thermal response are presented in Tables 3.1 and 3.2. For the outer node's dimensionless average temperature, Table 3.1 exhibits that the relative errors of approximate methods in comparison with the RK algorithm are quite small. The equivalent linearization method yields errors smaller than that of the Grande's approach. It is also seen from Table 3.2 that the dual criterion gives smaller errors than remaining methods.

Table 3.2. Outer node's dimensionless temperature amplitude χ with various values of thermal capacity C_2

C	χ_{RK}	χ_G	**Error (%)**	χ_{CL}	**Error (%)**	χ_{DC}	**Error (%)**
10000	0.0403	0.034924080	13.5187	0.034944460	13.4683	0.034944517	13.4681
12000	0.0373	0.032419844	13.2054	0.032439890	13.1517	0.032439941	13.1516
14000	0.0350	0.030537221	12.8108	0.030557158	12.7538	0.030557205	12.7537
16000	0.0331	0.029065587	12.3818	0.029085496	12.3218	0.029085539	12.3216
18000	0.0316	0.027877097	11.9483	0.027896997	11.8854	0.027897039	11.8853
20000	0.0303	0.026890788	11.5275	0.026910674	11.4620	0.026910713	11.4619
22000	0.0293	0.026053514	11.1286	0.026073369	11.0609	0.026073407	11.0607
24000	0.0283	0.025329259	10.7559	0.025349065	10.6861	0.025349102	10.6859
26000	0.0275	0.024692897	10.4104	0.024712637	10.3388	0.024712674	10.3387
28000	0.0268	0.024126433	10.0919	0.024146094	10.0187	0.024146129	10.0185
30000	0.0261	0.023616661	9.7989	0.023636230	9.7242	0.023636265	9.7241

3.4. Conclusions of Chapter 3

In this chapter, the author presents an extension of the dual criterion equivalent linearization method to find approximate solutions of a two-node thermal model of small satellites in Low Earth Orbit. Two important characteristics needed for the evaluation of temperature limits of satellite during its motion in orbit are average temperature and amplitude values. To get these quantities, a closed nonlinear system of equivalent linearization coefficients is established based on the proposed dual criterion, and then is solved by the Newton– Raphson iteration method. The main results obtained in the chapter can be summarized as follows:

- The graphs of evolutions of nodes in time obtained from the approximate methods (i.e. the Grande's approach, conventional and dual criterion linearization methods) are quite close to that obtained from the Runge–Kutta algorithm. This is clarified from the analysis of solution errors of analytical methods in comparison with the Runge– Kutta numerical solution.

- The efficiency of solution time of the proposed dual criterion method is recorded in the framework of two-node model in the problem of satellite thermal analysis.

- In the considered range of the thermal capacity from 10000 to 30000 JK^{-1}, the errors obtained from the proposed dual criterion for the average temperature and amplitude values are smaller than those obtained from the Grande's approach

The results of Chapter 3 are published in three papers [2], [5] and [6] in the *List of published works related to the author's thesis*.

CHAPTER 4. ANALYSIS OF THERMAL RESPONSE FOR SMALL SATELLITES IN LOW EARTH ORBIT USING MANY-NODE MODEL

4.1. Thermal analysis for solar array

In area of thermal control, the temperature specification for solar arrays of satellites is important because solar arrays supply main energy source for the operation of almost electrical devices and related equipment of satellites during motion in their orbits. The solar arrays are also composed of different materials. A solar array includes two surfaces: a front surface contains solar cells absorbed energy directly from solar rays; absorptivity coefficient of

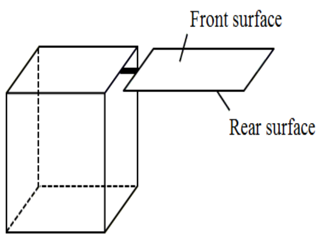

Figure 4.1. A model of solar array of a small satellite

the front surface is taken to be $\alpha_1 = 0.69$ whereas emissivity coefficient is $\varepsilon_1 = 0.82$; and a rear surface is coated by a material layer with absorptivity $\alpha_2 = 0.265$, and emissivity $\varepsilon_2 = 0.872$. In this section, to predict thermal responses of the solar array of the satellite, we use a model of two-node for front and rear surfaces. A model of the solar array is illustrated in Figure 4.1 (see [4]).

We will calculate thermal responses of the solar array in two cases:

The first case: The satellite always remains Earth-pointing attitude during motion (see Fig. 4.2 for the solar array only).

The second case: During the fraction of orbit while the satellite is illuminated, attitude of the satellite is controlled, so that the front surface (contains solar cells) always remains Sun-pointing attitude and is perpendicular to solar rays; during the eclipse period, rear surface remain Earth-pointing attitude (see Figure 4.3).

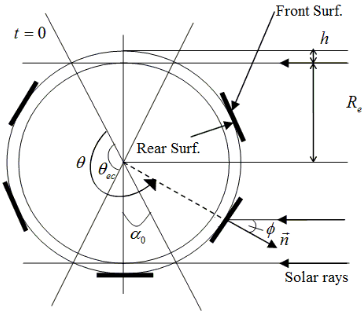

Figure 4.2. Earth-pointing attitude of the satellite in the first case

(*for the solar array only*)

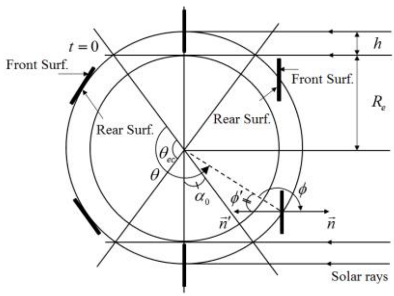

Figure 4.3. Attitude of the satellite in the second case

(*for the solar array only*)

We illustrate our calculations in the first case [calculation details for the second case can be seen in the full text of author's thesis]. In this case, we obtain temperature responses of two nodes (front and rear surfaces) as functions of time (see Fig. 4.4). It is seen that the obtained solutions appear almost periodic at the steady-state regime.

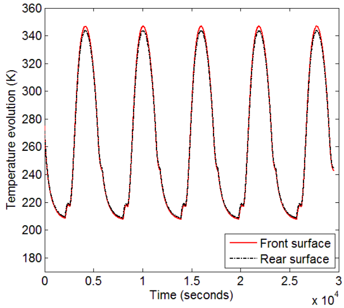

Figure 4.4. Temperature evolution of front and rear surfaces as functions of time

In this case, temperature values of the front surface are nearly close to those of the rear surface. This is because the solar array is a thin plate, the temperature difference between opposite flat surfaces is quite small.

4.2. Thermal analysis for box-shape satellite

We consider a box-shape satellite of size $L \times W \times H = 0.5 \times 0.5 \times 0.5\,(\mathrm{m}^3)$, thickness $\delta = 0.02$ (m) (Fig. 4.5), made from composite plate with the mass density $\rho = 158.90\,(\mathrm{kgm}^{-3})$; specific heat capacity $C_p = 883.70$ $(\mathrm{Jkg}^{-1}\mathrm{K}^{-1})$; material conductivity $\lambda = 5.39$ $(\mathrm{Wm}^{-1}\mathrm{K}^{-1})$; emissivity and absorbsivity of the material $\varepsilon = 0.82$, $\alpha = 0.65$, respectively.

The cover plates 1, 2, 3, 4, 5, 6 are numbered as shown in Fig. 4.5. Numbers 1 to 6 indicate that the satellite structure is separated into six-node with thermal characteristics assigned to each node.

The following sections, we will calculate the thermal response of nodes in two special trajectory cases when orbital angle $\beta = 0^0$ [the orbital plane is parallel to solar rays] and $\beta = 90^0$ [orbital plane is perpendicular to solar rays]. These two cases, namely, "Cold Case – CC" and "Hot Case – HC", are commonly used for satellite thermal analysis. In next section, we will analyze the thermal response of satellite structures in above cases.

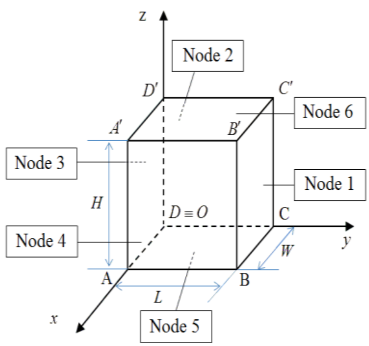

Figure 4.5. A model of a small box-shape satellite

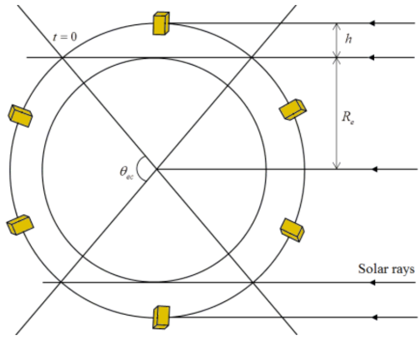

Figure 4.6. Earth-pointing attitude of the satellite in Cold Case

4.2.1. The Cold Case (CC)

In the CC, satellite's orbit is Sun-synchronous and orbital plane is parallel to solar rays. For simulation, we suppose that the satellite always remains Earth-pointing attitude during motion.

Table 4.1. The order of nodes in the thermal calculation in six-node model

Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
+Y	-X	-Y	+X	-Z	+Z

The order of nodes in thermal calculation is shown in Tab. 4.1. During motion, only four surfaces receive the thermal loadings from the space environment are +X, -X, +Z, -Z; also for other two sides

+Y and -Y, the applied thermal loadings are considered to equal zero. Temperature evolutions in time of six nodes of satellite are shown in Fig. 4.7.

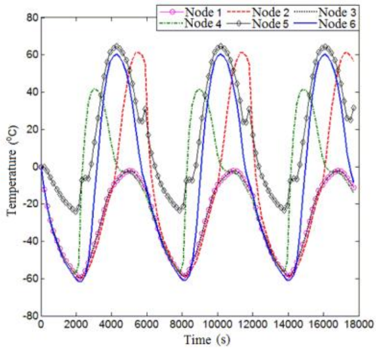

Figure 4.7. Temperature evolutions in time of six nodes of satellite in CC

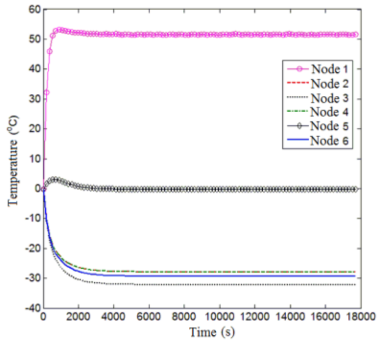

Figure 4.8. Temperature evolutions in time of six nodes of satellite in HC

4.2.2. The Hot Case (HC)

In this HC, surface +Y (node 1) always remains Earth-pointing attitude during motion. The thermal behavior of nodes is shown in Figure 4.8. Because thermal loadings are constant, after several periods of orbit, temperature values of nodes will tend to steady states and have constant values.

4.3. Thermal analysis for box-shape satellite with a solar array

A box-shape satellite with a solar array can be modeled as a system with different lumped thermal nodes. We use an eight-node model to estimate temperatures at nodal elements i.e. six nodes for cover plates, and two nodes for front and rear surfaces of the solar array (as shown in Fig. 4.9). This model is a simplified one and will be a basis for exploring the more complex satellite model.

In the thesis, the author calculates thermal loadings and analyzes thermal response of nodes in three cases of orbital configuration: Cold-Case, Hot-Case 1 (i.e. Hot-Case for the satellite body), Hot-Case 2 (i.e. Hot-Case for the solar array). The nodal order in thermal calculation layout is shown in Tab. 4.2.

Table 4.2. The nodal order in thermal calculation layout in eight-node model

Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7	Node 8
+Y	-X	-Y	+X	-Z	+Z	Rear surface	Front surface

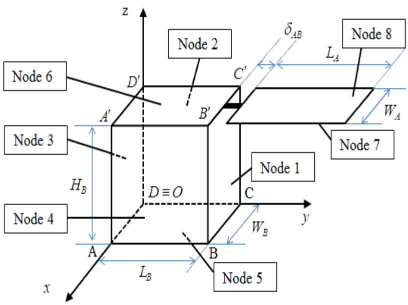

Figure 4.9. A model of a small satellite with a solar array

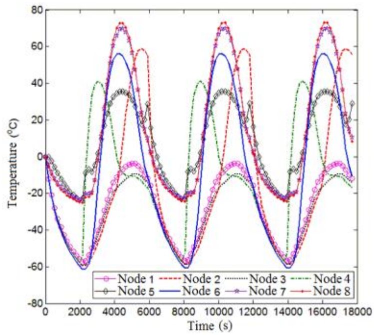

Figure 4.10. Temperature evolutions in time of eight nodes of satellite in CC

We here illustrate calculation results in the Cold-Case. Temperature values of nodes in time will be obtained as we solve the thermal balance equations of nodes (see Figure 4.10). It is seen that the predicted temperatures of the satellite obtained from our numeral analysis are within the allowable temperature limit of satellite. In this case, the effects of material properties such as absorbtivity and emissivity on the thermal responses of nodes are explored (see [3] in detail).

4.4. Conclusions of Chapter 4

In this Chapter 4, the author has studied thermal models of satellite structure and obtained the following main results:

- Models of thermal loadings from space environment are established in the framework of Low Earth Orbit.

- Simplified models (i.e. two-node model for solar arrays, six-node-model for the box-shape satellite and eight-node model for another box-shape satellite with a solar array) are constructed based on the geometrical dimensions and material properties of satellite.

- The temperature evolutions in time of nodes are obtained using the Runge-Kutta algorithm to solve thermal balance equations.

- The maximum and minimum temperature information of nodes shows that the predicted temperatures of the satellite obtained from our numeral analysis are within the allowable temperature limit range of satellite.

The results of Chapter 4 are published in three papers [3], [4] and [8] in the *List of published works related to the author's thesis*.

CONCLUSIONS

This thesis presents new and important findings in thermal analysis of satellites based on single-node, two-node and many-node thermal models. For single-node and two-node models, the author has applied analytical methods including the equivalent linearization method and Grande’s linearization approach to find approximate responses of thermal models; and then investigated qualitative behaviors of the solution depending on the system parameters. For many-node models, the author has used a fourth-order Runge-Kutta method to compute solutions and examine the basic characteristics of nodal temperatures in thermal models with different trajectories and has indicated the suitability of the predicted temperature in the allowable temperature limit range of satellite components.

The new findings of the thesis

The thesis has achieved the following new results:

- It is the first time, in the area of satellite thermal analysis, the method of equivalent linearization using different criteria including mean-square and dual ones is applied to find approximate temperature responses of small satellites in Low Earth Orbit.

- The author has developed analytical methods for single-node and two-node models based on the proposed dual criterion in the framework of nonlinear problem of the satellite thermal balance equations.

- Numerical results of the thermal behavior analysis show that the dual criterion yields higher accuracy than those obtained from the Grande's approach.

- Simplified models of thermal loadings and satellite thermal structures are constructed and developed for small satellites in Low Earth Orbit. This result of thermal analysis plays a fundamental role for purpose of designing and calculating more complex satellite thermal models.

Suggestions

- Developing and extending the dual criterion equivalent linearization method to investigate thermal responses of satellites with different case of external loadings in which random factors are taken into account in thermal models.

- Developing different satellite thermal models including geometrical configuration, material models, thermal loading models, towards the construction of a specialized software for satellite thermal analysis.

LIST OF PUBLISHED WORKS RELATED TO THE AUTHOR'S THESIS

1. Nguyen Dong Anh, Nguyen Nhu Hieu, Pham Ngoc Chung, Nguyen Tay Anh (2016), Thermal radiation analysis for small satellites with single-node model using techniques of equivalent linearization, *Applied Thermal Engineering*, 94, pp. 607-614. (SCI-E journal)
2. Pham Ngoc Chung, Nguyen Nhu Hieu, Nguyen Dong Anh, Dinh Van Manh (2017), Extension of dual equivalent linearization to nonlinear analysis of thermal behavior of a two-node model for small satellites in Low Earth Orbit, *International Journal of Mechanical Sciences*,133, 513–523. (SCI journal)
3. Pham Ngoc Chung, Nguyen Dong Anh, Nguyen Nhu Hieu (2017), Nonlinear analysis of thermal behavior for a small satellite in Low Earth Orbit using many-node model, J*ournal of Science and Technology Development Vietnam National University-HCM City*, 20, pp. 66-76 (ISSN 1859-0128). (National journal)
4. Pham Ngoc Chung, Nguyen Nhu Hieu, Nguyen Dong Anh (2016), Thermal radiation analysis for solar arrays of a small satellite in Low Earth Orbit, *The 4th international Conference on Engineering Mechanic and Automation (ICEMA4)*, pp 146-153.
5. Nguyen Dong Anh, Nguyen Nhu Hieu, Pham Ngoc Chung (2013), Analysis of thermal responses for a satellite with two-node model using the equivalent linearization technique, *International Conference on Space*, *Aeronautical*, *and Navigational Electronics*, Vol. 113(335), pp. 109-114.
6. Nguyen Nhu Hieu, Nguyen Đong Anh, Pham Ngoc Chung (2014), Phương pháp giải tích trong bài toán mô hình nhiệt hai nút của vệ tinh nhỏ trên quỹ đạo thấp, *Conference on "Research and application of space technology", Hanoi, 2014, Natural Science and Technology Publishing House, ISBN:978-604-913-305-3*, pp 469-479.
7. Pham Ngoc Chung, Nguyen Đong Anh, Nguyen Nhu Hieu, Phan Thi Tra My (2015), Nghiên cứu giải tích ứng xử nhiệt của vệ tinh nhỏ trên quỹ đạo thấp dựa theo mô hình một nút, *Proceedings of the Conference on Technical Mechanics, Da Nang, 8/2015*, pp 11-18.
8. Nguyen Nhu Hieu, Vu Lam Đong, Nguyen Đong Anh, Nguyen Đinh Kien, Pham Ngoc Chung (2015), Phân tích dao động, độ bền, ổn định và nhiệt của kết cấu vệ tinh nhỏ trên quỹ đạo thấp của trái đất, *Space science and technology program (2012-2015)*, pp 71-104, ISBS:978-604-913-498-2.