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RESEARCH FOR ORTHOGONALITY DEVELOPING APPLIED IN THE ANALYSIS OF STABILITY AND NONLINEAR OSCILLATION

Major: Engineering mechanics
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SUMMARY OF DOCTORAL THESIS

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PREFACE

1. The necessities of the thesis

In order to improve the accuracy of first-order approximation solutions of Galerkin's method as well as develop orthogonal properties and apply to some typical problems of elastic stability of columns and consider with constant cross-section and cross-sectional change, while improving the orthogonality by developing a dual-weighted linearization method for nonlinear deterministic dynamic systems by the equivalent replacement of a nonlinear function with a linearity is briefly corrected using a dual method with two forward and backward substitutions. The selection of an additional a weight parameter connecting the two objective functions is investigated and applied to the frequency analysis of the nonlinear oscillations, and some case studies are then performed to verify the accuracy and the effect of non-linearity on the efficiency of the proposed technique.

With the above analysis, the author has chosen the topic: "***Research for orthogonality developing applied in the analysis of stability and nonlinear oscillation***" to be the research topic.

2. Research objectives of the thesis

Developing the orthogonality of the weighted residuals method (in particular the Galerkin method) with the normal mean and applying the weighted dual criterion of the equivalent linearization method to the elastic stability problems.

Solve the problem of choosing a specific weight function among a class of single-parameter weight functions and develop orthogonality by proposing to build a new weighted local average applied to Galerkin and combined with the method of least squares to investigate the stability problem with columns with constant cross section and variable cross section.

Proposing a new and efficient alternative calculation tool for engineering calculations in the design of structural systems with variable cross-sections.

Building process, developing orthogonal property through single-weight dual linearization method for nonlinear deterministic dynamic systems and applying to frequency analysis of nonlinear oscillations in some cases typical and compare the results of some approximate methods to evaluate the effectiveness of the proposed method.

3. Contents of the thesis

The thesis consists of an introduction, 04 chapters, a conclusion, a list of the author's published works related to the thesis, and references. The main contents of the chapters are as follows:

Chapter 1: "***Overview of methods to solve elastic stability problems and one-degree-of-freedom system of nonlinear oscillations***". Presenting stability models, elastic stability problems, one-degree-of-freedom nonlinear oscillation problems and some common nonlinear oscillation systems, and an overview of methods to solve stability problems and nonlinear oscillations problems, approximation methods of the weighted residual method, Galerkin and orthogonal analysis in these methods.

Chapter 2: "***Equivalent linearization method***", present equivalent linearization methods and analyze the advantages and disadvantages of the methods, the idea of weighted dual linearization for stability problem and nonlinear oscillation problem as a basis for orthogonal property development and applied in Chapter 3, Chapter 4.

Chapter 3: "***Develop orthogonality applied in stability problem analysis***". Developing orthogonality through weighted duality criterion (WDC) and by weighted local averaging (WLA) at a local value r with a local function $r^{f(r)}$. Applying weighted dual linearization, GWLA, SGWLA, choosing weight function, numerical survey with elastic stability problems of constant cross-section, variable cross-section, concluding about the effectiveness of the proposed technique.

Chapter 4: "***Develop orthogonality applied in the analysis of nonlinear oscillation problems***". Building the theoretical basis, developing the orthogonal property, building a linearization process and numerical investigation with nonlinear deterministic oscillation problems and concluding on the efficiency of the proposed computational technique.

Conclusion and new contributions of the thesis and future research directions. List of published works and citations.

CHAPTER 1. OVERVIEW OF METHODS TO SOLVE ELASTIC STABILITY PROBLEMS AND ONE-DEGREE-OF-FREEDOM SYSTEM OF NONLINEAR OSCILLATIONS

1.1. Elastic stability

1.1.1. Static method

Method of setting and solving differential equations; Initial parameter method; force method; Transposition method; Mixed method; Finite difference method; The leash method; The test method is correct at each point; Bubnov-Galerkin method; Correct solution method. In practice, applying static methods to find exact solutions of stability problems is often difficult and sometimes impossible.

1.1.2. Dynamic method

Set up and solve the system's partial oscillation equations. Determine the critical force by arguing the solution property of motion: if the system's oscillation amplitude increases continuously with time, the initial equilibrium form is unstable; conversely, if the system is always oscillating slightly around the initial equilibrium position or decreasing, then the form is stable.

1.1.3. Energy method

Directly the Lejeune-Dirichlet principle; Rayleigh-Ritz method; Timoshenko method. Due to the presumption of the system's deformation, the resulting critical force is usually approximate and gives results larger than the exact critical force value.

The paths of the three types of methods (static method; dynamic method; energetic method) although different, give the same result for the conserved system. For non-conservative systems, static methods and energetic methods lead to inaccurate results, one must use dynamical methods.

1.2. Instability models

The first type of instability that has been studied and received a lot of attention is class 1 or classical or branching instability. Type 2 instability (snapthrough buckling) is a phenomenon characterized by a visible and abrupt jump from an equilibrium to another equilibrium with a displacement greater than the displacement at the initial equilibrium. Another type of instability that Libove [25] calls finite displacement instability.

1.3. Elastic stability problems

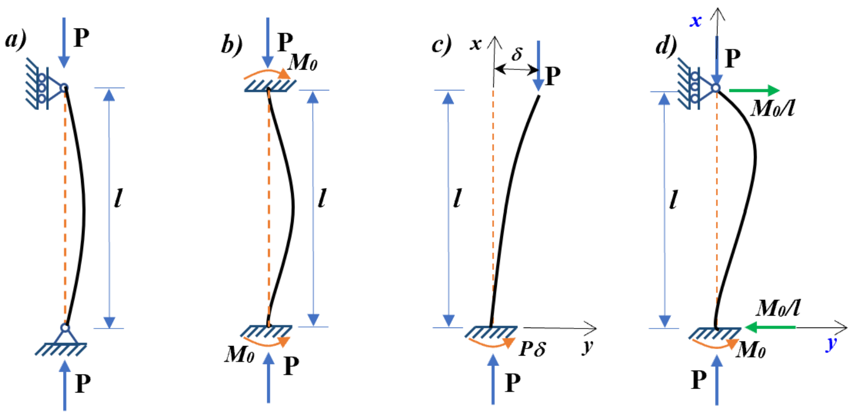


Figure 1.6. Eulerian stability problems

1.3.1. Column with two hinged ends (P-P)

1.3.2. Column with two fixed ends (C-C)

1.3.3. Column with one fixed end and one free end (C-F)

1.3.4. Column with a fixed end and a hinge end (C-P)

1.4. Nonlinear oscillation problem

1.4.1. Classification of Nonlinear Vibrations: Free oscillation, damping oscillation, undamping oscillation, resisted oscillation, predetermined oscillation, stochastic oscillation, 1-degree-of-freedom oscillation, multiple degrees of freedom, and infinite degrees of freedom

1.4.2. One-degree-of-freedom nonlinear mechanical system:

$$m\ddot{x} + b_{tt}\dot{x} + k_{tt}x + \mathrm{g}_{pt}(x, \dot{x}) = u(t) \qquad (1.5)$$

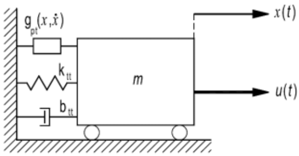


Figure 1.7. One-degree-of-freedom nonlinear mechanical system

1.4.3. Some common nonlinear oscillation systems: Lutes Sarkani, Van der pol oscillator system, with third-order nonlinear damping, exponential nonlinear elasticity, Duffing, Atlik-Utku, with fractional order restoring force, Duffing harmonic, capable of organic expansion term, Duffing style oscillator system.

1.5. Some approximate methods to solve differential equations

1.5.1. Rayleigh–Ritz Variation Method

1.5.2. Weighted Residual Method

$$\int W_i R_\Gamma d\Gamma = 0 \qquad (1.34)$$

The orthogonality shown in Equation (1.34) and the different weighted residual methods in the function definition W_i.

1.5.3. Galerkin method and orthogonality of residuals with comparison functions

The Galerkin method solves directly from differential equations while the Rayleigh-Ritz method focuses on energy.

$$\int_0^l Q(\phi)g_i(x)\,dx = 0, \text{với i} = 1, 2, \dots, \text{n} \tag{1.49}$$

The relationships given by equation (1.49) are called Galerkin equations. For a given problem any hypothetical displacement function that fits the boundary conditions and the Galerkin equation will be an approximate solution of the problem. This equation shows the orthogonality between the n trial functions and the residual.

1.6. Conclusion Chapter 1

Presents an overview of methods to solve elastic stability problems and exact solutions of these problems, and also shows the general properties of approximation methods such as Rayleigh-Ritz, Galerkin and other methods. The weighted residual method (MWR) is shown by the orthogonality between the residuals and the comparison function, and the orthogonality depends on the averaging operator acting on the residuals. Therefore, to obtain other approximations using the same Galerkin procedure and comparison functions, we can modify the averaging operator to be the orthogonal property. The thesis builds and develops the orthogonal property in analyzing the problem of stability and nonlinear oscillation based on the exact results obtained from the problem of stability and nonlinear free oscillation to compare with the method Galerkin and other approximate methods to evaluate the effectiveness of the proposed method.

CHAPTER 2. EQUIVALENT LINEARIZATION METHOD

2.1. Introduce

This chapter presents the equivalent linearization methods for the one-degree-of-freedom system in Section 2.2 with linearization coefficients, we have different versions based on the classical criteria, criteria of minimum potential error, equivalent linearization criteria with adjustment, partial linearization criteria. The development of the duality criterion and its application is presented in 2.3. The formulation and idea of the weighted dual criterion and its properties are detailed in 2.4. Conclusion Chapter 2 presented in 2.5.

2.2. Equivalent linearization method for one-degree-of-freedom system

2.3. The dual criterion of the equivalent linearization method

2.4. Weighted dual criterion

$$S_d = (1-p)\langle (A - k_d B)^2 \rangle + p\langle (k_d B - \lambda_d A)^2 \rangle \longrightarrow \min_{k_d, \lambda_d} \tag{2.65}$$

The criterion (2.65) is called the weighted dual criterion (WDC), which is stated as the weighted average of the mean squares of the forward and backward substitution processes that are the smallest in terms of the linearization coefficients k_d and the return coefficient λ_d given by (2.60), (2.61) and r^2 are defined and given by (2.62) with (2.63) and the condition to be satisfied from (2.64)

2.5. Conclusion of chapter 2

The equivalent linearization method is how to find the linearization coefficients for a given nonlinear system. The goal of the improvements and developments to the equivalence linearization method is to determine the linearization coefficients so that an approximate mean squared or variance can be obtained that is closest to the exact solution. The development of orthogonality in averaging applied in the weighted duality criterion for stability problems will be examined in detail in Chapter 3, and the extension and development of orthogonality from Galerkin's method with weighted local mean. In Chapter 4, the author will present the development of orthogonality in dual-equivalent linearization with a weight parameter for nonlinear deterministic dynamic systems, at the same time, survey and apply dual equivalence linearization with selected weight parameters to analyze nonlinear free oscillation frequencies and some typical cases and evaluate the effectiveness of the proposed method.

CHAPTER 3 DEVELOP ORTHOGONALITY APPLIED IN STABILITY PROBLEM ANALYSIS

3.1. Introduce

This chapter is organized as follows: Section 3.2 Applying the weighted dual criterion to the elastic stability problem, Choosing a weight parameter p presented in Section 3.3 and Section 3.4 presents the development of orthogonality through weighted local averaging. Then the combination of weighted local averaging with the Galerkin method is given in Section 3.5. The accuracy of the Galerkin method with its weighted local averaging and its approximation is investigated in Section 3.6 with the application of determining the critical load of a constant section elastic Euler column and with different typical boundary conditions. In addition, the application of the Galerkin method with weighted local averaging for columns with variable cross-section is presented in Section 3.7, which develops the orthogonality in the least squares method. deployed. Finally, a discussion of the orthogonal development results is summarized in Section 3.8, followed by the conclusion of Chapter 3 in 3.9.

3.2. Applying the dual criterion to the problem of elastic stability

3.2.1. Column with two hinged ends (P-P)

Table 3.1. Critical force P_{cr}^{G} and error for different functions f_i

No	f_i	Galerkin's solution	Error with Euler solution (%)
1	$x(l-x)$	10	1,321180
2	$x(l-x)^2$	14	41,849660
3	$x^2(l-x)$	14	41,849660
4	$x^2\ (l-x)^2$	12	21,585420
5	$x(l-x)+x^2(l-x)$	10.5	6,387240
6	$x(l-x)+x^2(l-x)^2$	9,87097	0,013814

Table 3.2. Critical force P_{cr}^{dn} and error for $y(x) = x(l - x)$

t	v	r	p	P_{cr}^{dn}	$Error\ for\ P_{cr}^{E}$
1	0	0,91287092	0,087129080	9,84341568	-0,265347%
1	1		0,079537600	9,85802719	-0,117302%
1	2		0,072607600	9,87119382	0,016104%
2	2		0,006326230	9,98940041	1,213787%
2	1		0,006930040	9,98838285	1,203477%
3	1		0,000603808	9,99899315	1,310982%
3	2		0,000551198	9,99908091	1,311871%
2	3		0,005775030	9,99032840	1,223190%
1	3		0,066281300	9,88307270	0,136462%
3	3		0,000503173	9,99916103	1,312683%
4	3		0,000043841	9,99992693	1,320443%
3	4		0,000459332	9,99923415	1,313424%
4	4		4,00212E-05	9,99993330	1,320508%
5	5		3,18319E-06	9,99999469	1,321130%
6	6		2,53183E-07	9,99999958	1,321179%
10	10		1,01327E-11	10,0000000	1,321184%

Table 3.3. Critical force P_{cr}^{dn} and error for $y(x)$

	$y(x)$	r	p	P_{cr}^{dn}	P_{cr}^{E}	$t;\ v$	P_{cr}^{G}
1	$x^2(l-x)$	0,68313	0,316869	11,22346	9,8696	1; 0	14,0000
			0,216463	12,20212	9,8696	1; 1	
2	$x^2(l-x)^2$	0,53452	0,132994	10,81502	9,8696	1; 2	12,0000
			0,115814	10,97333	9,8696	2; 1	
			0,465478	7,39818	9,8696	1; 0	
3	$x(l-x)+x^2(l-x)$	0,83666	0,163339	9,91905	9,8696	1; 0	10,5000
			0,136660	10,02398	9,8696	1; 1	
4	$x(l-x)+x^2(l-x)^2$	0,99942	0,000576	9,87096	9,8696	1; 0	9,87097
			0,000575	9,87096	9,8696	1; 1	
			0,000575	9,87096	9,8696	1; 2	

3.2.2. Column with two fixed ends (C-C)

P_{cr}^{E}	P_{cr}^{G}	P_{cr}^{dn}
$39{,}4784\mathrm{EI}/l^2$	$41{,}3369\mathrm{EI}/l^2$	$39.3943\ \mathrm{EI}/l^2$

3.2.1. Column with one fixed end and one free end (C-F)

P_{cr}^{E}	P_{cr}^{G}	P_{cr}^{dn}

2,4674 EI/l^2	1,33EI/l^2	1,5037 EI/l^2

3.2.1. Column with a fixed end and a hinge end (C-P)

P_{cr}^{E}	P_{cr}^{G}	P_{cr}^{dn}
20,1906 EI/l^2	22,4211EI/l^2	21,5968 EI/l^2

3.3. Choosing weight parameter *p* for elastic stability problem

$$p = (1 - r^t)^u . r^v \tag{3.41}$$

Table 3.4. Critical force P_{cr}^{dn} and error for f_3 with weight functions $p_i(r)$

i	p_{i2};p_{i0}	k_{i2};k_{i0}	P_{cr}^{dn} và error *(%)*	P_{cr}^{G} và *%)*
1	0,124145; 0,10680	-10,6745; 10,6586	21,3330 (*5,91297*)	
2	0,056938; 0,03302	-11,0621; 10,9371	21,9992 (*9,22042*)	22,4211
3	0,091341; 0,08876	-10,8674; 10,7295	21,5968 *(7.22262)*	(*11,3147*)
4	0,067206; 0,07377	-11,0048; 10,7870	21,7917 *(8,19021)*	
5	0,018898; 0,00984	-11,2689; 11,0187	22,2876 *(10,6512)*	

Table 3.5. Critical force P_{cr}^{dn} and error for f_j with $p_i(r)$

j	f_j	p_i	P_{cr}^{dn} và error *(%)*	P_{cr}^{G} và *(%)*
1	$f_1 = \frac{3M_0}{4lEI} x^2(l - x)$	p_1	19,8803 (*-1.29959)*	21,0000 *(4.2595)*
		p_2	20,4618 *(1,58733)*	
		p_3	20,2217*(0,39551)*	
		p_4	20,4509*(1,53348)*	
		p_5	20,8157*(3,34441)*	
2	$f_2 = \frac{3M_0}{4lEI} x^3(l - x)$	p_1	31,0252 *(54,0319)*	32,8000 *(62,8434)*
		p_2	31,7871 *(57,8144)*	
		p_3	31,6788 *(57,2771)*	
		p_4	32,0766 *(59,2520)*	
		p_5	32,4233 *(60,9733)*	
4	$f_4 = \frac{3M_0}{4lEI} x^4(l - x)$	p_1	44,5491 *(121,174)*	47,1429 *(134,052)*
		p_2	45,6020*(126,402)*	
		p_3	45,5307 *(126,048)*	
		p_4	45,9478 *(128,119)*	
		p_5	46,5097 *(130,908)*	
5	$f_5 = \frac{M_0}{10lEI} x^2(7l^2 - 2lx - 2x^2)(l - x)$	p_1	20,8242 *(3,38678)*	21,8042 *(8,25201)*
		p_2	21,4487*(6,48699)*	
		p_3	21,0456 (*4,48593)*	

		p_4	21,2132 *(5,31816)*	
		p_5	21,6940*(7,70495)*	
6	$f_6 = \frac{M_0}{4lEI} x^4(3l-2x)(l-x)$	p_1	31,4423*(56,1026)*	33,1190 *(64,4274)*
		p_2	32,3490*(60,6042)*	
		p_3	31,9289*(58,5186)*	
		p_4	32,2642 *(60,1832)*	
		p_5	32,8613*(63,1479)*	
7	$f_7 = \frac{M_0}{10lEI} x(7l^3x^2 - 9x^3l^2 + 2x^5)$	p_1	33,2967 *(65,3092)*	34,5291 *(71,4279)*
		p_2	34,1035 *(69,3149)*	
		p_3	33,5610 *(66,6215)*	
		p_4	33,7619 *(67,6192)*	
		p_5	34,3975 *(70,7747)*	

3.4. Weighted local averaging

$$\langle g(x)|x^{1+\alpha-2\alpha r}, r\rangle = \frac{r^{1+\alpha-2\alpha r}}{r}\int_0^r g(x)dx + \frac{1-r^{1+\alpha-2\alpha r}}{1-r}\int_r^1 g(x)dx \qquad (3.53)$$

3.5. Developing orthogonality in Galerkin's method with weighted local averaging

Applying GCA to (3.57) leads to the following equations of the orthogonal condition:

$$\left\langle A\left(\sum_{j=1}^{N} a_j W_j(x)\right) W_i(x)\right\rangle = 0, i = 1,2,\ldots,N \qquad (3.59)$$

$$\frac{r^{1+\alpha-2\alpha r}}{r}\int_0^r A\left(\sum_{j=1}^{N} a_j W_j(x)\right) W_i(x)dx + \frac{1-r^{1+\alpha-2\alpha r}}{1-r}\int_r^1 A\left(\sum_{j=1}^{N} a_j W_j(x)\right) W_i(x)dx = 0, \quad i = 1,2,\ldots,N \qquad (3.61)$$

or

$$\sum_{j=1}^{N} a_j \left[\frac{r^{1+\alpha-2\alpha r}}{r} \int_0^r A\left(W_j(x)\right) W_i(x) dx + \frac{1-r^{1+\alpha-2\alpha r}}{1-r} \int_r^1 A\left(W_j(x)\right) W_i(x) dx\right] = 0, \quad i = 1,2,\ldots,N \tag{3.66}$$

GWLA allows getting much more precise solutions to some problems GWLA will be deployed to find the critical loads for elastic columns with different types of boundary conditions and cross-sections, when compared compared with solutions obtained from GCA.

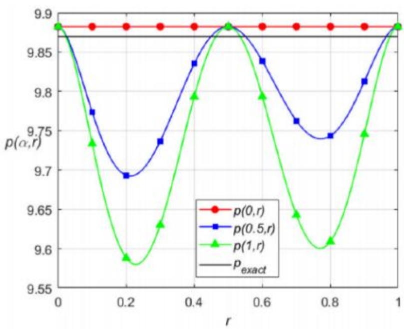
3.6. Elastic instability of Euler column with constant cross-section

$$P_{GWLA}^{const}(\alpha,r) = -\frac{\frac{r^{1+\alpha-2\alpha r}}{r}\int_0^r \frac{d^4w}{dx^4} w dx + \frac{1-r^{1+\alpha-2\alpha r}}{1-r}\int_1^1 \frac{d^4w}{dx^4} w dx}{\frac{r^{1+\alpha-2\alpha r}}{r}\int_0^r \frac{d^2w}{dx^2} w dx + \frac{1-r^{1+\alpha-2\alpha r}}{1-r}\int_1^1 \frac{d^2w}{dx^2} w dx} \tag{3.72}$$

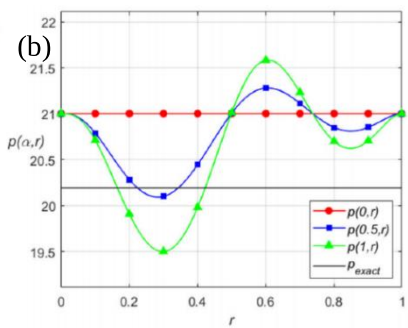
To reduce the computational volume, the author proposes a simplified Galerkin method with weighted local averaging (SGWLA for short).

$$P_{SGWLA}^{const} = min\left(P_{GWLA}^{const}(0.5,0.25), P_{GWLA}^{const}(0.5,0.75)\right) \tag{3.79}$$

(a)



(b)



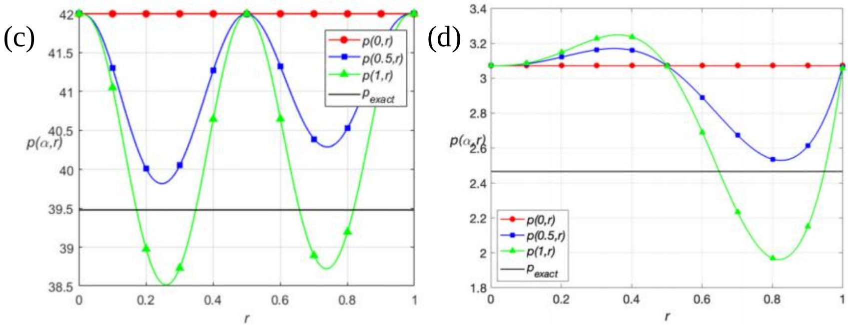


Figure 3.2. Normalized critical load, $P_{GWLA}^{const}(\alpha, r)$ by GWLA, as a function of r, $r \in [0,1]$ and three values of α: 0, 0,5 and 1 compare with exact value, P_{exact}^{const}, for a column of constant cross-section: a) P-P, b) C-P, c) C-C và d) C-F.

Table 3.7. Normalized critical loads obtained by GWLA and SGWLA, GCA and exact solutions for four different column types of constant cross-section.

Cột	P_{exact}^{const}	P_{GWLA}^{const}	%E	P_{SGWLA}^{const}	%E	P_{GCA}^{const}	%E
P-P	9,8696	9,7054	1,6640	9,7011	1,7074	9,8823	0,1289
C-P	20,1907	20,1444	0,2293	20,1247	0,3271	21,0000	4,0081
C-C	39,4784	39,9657	1,2344	39,8185	0,8614	42,0000	6,3872
C-F	2,4674	2,5248	2,3263	2,5881	4,8900	3,0725	24,5230

3.7. Elastic instability of Euler columns with variable cross-section

3.7.2. Convert the variable section column to the equivalent column

$$P_{GWLA}^{var} = k^{var} P_{GWLA}^{const} \tag{3.93}$$

3.7.2. Applied to the problem Euler with variable cross-section

For columns with exponential moment of inertia, we have

$$G(Lx) = e^{-aLx} \tag{3.94}$$

Table 3.9. Normalized critical load, obtained by GWLA, GCA and exact solution for four column types with exponential moment of inertia

Cột	aL	p_{exact}^{exp}	p_{GCA}^{exp}	$\%E$	$p_{GWLA}^{exp,CA}$	$\%E$	$p_{GWLA}^{exp,WLA}$Er	$\%E$
P-P	0	9,8696	9,8823	0,1289	9,7054	1,6639	9,7054	1,6639

	0,1	9,3800	9,4020	0,2349	9,2337	1,5597	9,2210	1,6951
	0,5	7,6340	7,7308	1,2676	7,5923	0,5462	7,5386	1,2497
	1	5,8270	6,1017	4,7139	5,9925	2,8402	5,9043	1,3266
C-P	0	20,1907	21,0000	4,0081	20,1444	0,2293	20,1444	0,2294
	0,1	19,2000	20,1519	4,9577	19,3269	0,6609	19,1603	0,2068
	0,5	15,6400	17,2408	10,2355	16,5210	5,6330	15,7930	0,9783
	1	11,9900	14,4716	20,6976	13,8517	15,5271	12,6102	5,1726
C-C	0	39,4784	42,0000	6,3872	39,9657	1,2344	39,9657	1,2343
	0,1	37,5500	39,9778	6,4655	38,0322	1,2842	37,6781	0,3411
	0,5	30,6000	33,2473	8,6514	31,5969	3,2578	30,0830	1,6895
	1	23,4900	27,1709	15,6700	25,7867	9,7773	23,2644	0,9604
C-F	0	2,4674	3,0725	24,5230	2,5248	2,3263	2,5248	2,3263
	0,1	2,3943	2,1381	10,6993	2,4588	2,7014	2,4476	2,2268
	0,5	2,1104	-0,6757	132,0193	2,2212	5,2558	2,1702	2,8320
	1	1,7824	-2,7134	252,2319	1,9731	10,7036	1,8825	5,6155

For columns whose moment of inertia is given by a power function,

$$G(Lx) = (1 - bLx)^a \tag{3.101}$$

$$P_{GWLA}^{pow,CA} = k_{CA}^{pow} P_{GWLA}^{const} \tag{3.103}$$

$$P_{GWLA}^{pow,WLA} = k_{WLA}^{pow} P_{GWLA}^{const} \tag{3.104}$$

Table 3.12. Normalized critical loads obtained by GWLA, GCA and exact solutions for four column types with linearly varying moment of inertia (a = 1)

Cột	bL	P_{exact}^{pow}	P_{GCA}^{pow}	$\%E$	$P_{GWLA}^{pow,CA}$	$\%E$	$P_{GWLA}^{pow,WLA}$	$\%E$
P-P	0	9,8696	9,8823	0,1289	9,7054	1,6639	9,7054	1,6639
	0,1	9,3720	9,3882	0,1729	9,2201	1,6205	9,2069	1,7613
	0,3	8,3430	8,4000	0,6828	8,2496	1,1196	8,2099	1,5954
	0,5	7,2560	7,4117	2,1462	7,2791	0,3177	7,2130	0,5932
C-P	0	20,1907	21,0000	4,0081	20,1444	0,2293	20,1444	0,2294
	0,1	19,1700	20,1250	4,9817	19,3010	0,6831	19,1287	0,2153
	0,3	17,0300	18,3750	7,8978	17,6143	3,4308	17,0972	0,3944
	0,5	14,7400	16,6250	12,7883	15,9274	8,0554	15,0656	2,2089
C-C	0	39,4784	42,0000	6,3872	39,9657	1,2344	39,9657	1,2343
	0,1	37,4800	39,9000	6,4568	37,9578	1,2749	37,5889	0,2907
	0,3	33,2700	35,7000	7,3039	33,9421	2,0201	32,8354	1,3062
	0,5	28,7000	31,5000	9,7561	29,9259	4,2715	28,0819	2,1537
C-F	0	2,4674	3,0725	24,5230	2,5248	2,3263	2,5248	2,3263

	0,1	2,3932	2,0835	12,9424	2,4576	2,6926	2,4458	2,1968
	0,3	2,2350	0,1054	95,2830	2,3231	3,9438	2,2879	2,3668
	0,5	2,0623	-1,8726	190,8021	2,1887	6,1302	2,1300	3,2814

Table 3.13. Normalized critical loads obtained by GWLA, GCA and exact solutions for four column types with linearly varying moment of inertia (a = 2)

Cột	bL	P_{exact}^{pow}	P_{GCA}^{pow}	$\%E$	$P_{GWLA}^{pow,CA}$	$\%E$	$P_{GWLA}^{pow,WL}$	$\%E$
P-P	0	9,8696	9,8823	0,1289	9,7054	1,6639	9,7054	1,6639
	0,1	8,8930	8,9223	0,3297	8,7626	1,4661	8,7373	1,7509
	0,3	7,0050	7,1717	2,3801	7,0433	0,5469	6,9739	0,4438
	0,5	5,1980	5,6470	8,6381	5,5460	6,6941	5,4413	4,6812
C-P	0	20,1907	21,0000	4,0081	20,1444	0,2293	20,1444	0,2294
	0,1	18,1900	19,3050	6,1297	18,5107	1,7630	18,1779	0,6652
	0,3	14,2900	16,4250	13,6809	15,5612	8,8954	14,6345	2,4108
	0,5	10,5300	13,6250	29,3922	13,0356	23,7953	11,6112	10,2681
C-C	0	39,4784	42,0000	6,3872	39,9657	1,2344	39,9657	1,2343
	0,1	35,5600	37,9600	6,7492	36,1026	1,5259	35,3956	0,4623
	0,3	27,9100	30,8400	10,4980	29,2949	4,9619	27,3545	1,9903
	0,5	20,4800	25,0000	22,0703	23,7109	15,7757	20,7798	1,4637
C-F	0	2,4674	3,0725	24,5230	2,5248	2,3263	2,5248	2,3263
	0,1	2,3190	1,2076	47,9254	2,3933	3,2022	2,3703	2,2114
	0,3	2,0120	-1,8431	191,606	2,1471	6,7155	2,0824	3,4967
	0,5	1,6830	-3,9885	336,989	1,9238	14,3061	1,8222	8,2724

3.8. Discussion on the results of developing orthogonal properties based on the WLA technique

3.9. Conclusion of chapter 3

These new numerical algorithms can provide a new and efficient alternative to engineering calculations in the design of structural systems with variable cross-sections. However, more comprehensive studies need to be carried out to find suitable weight functions that can give the best approximations to larger classes of problems. In particular, the proposed WLA can be verified for columns with other boundary conditions and is also extended to the buckling problems of more complex structures such as columns, plates and nonlinear shells where using of the analytic form of the Galerkin method is often limited to first-order approximations.

CHAPTER 4. DEVELOP ORTHOGONALITY APPLIED IN STABILITY PROBLEM ANALYSIS

4.1. Introduce

This chapter develops orthogonality through dual equivalence linearization with a weight parameter for nonlinear deterministic dynamic systems. The problem of replacing the equivalence of a nonlinear function with a linear function is briefly modified using a dual method consisting of two steps forward and backward. The additional selection of a weight parameter connecting the two objective functions is investigated using semi-analytic analysis. The dual equivalent linearization with the weight parameter was chosen for frequency analysis of the nonlinear free oscillations, and some specific case studies were then performed to verify the accuracy and influence of nonlinearity on the efficiency of the proposed technique.

The content of the chapter focuses on the development of dual equivalent linearization (DEL) with a weight parameter for nonlinear deterministic dynamic systems. Furthermore, choosing a weight parameter connecting two objective functions is the main goal. This chapter is structured as follows: Section 4.2 presents the formula of the DEL, its basic properties and the selection of proposed weights using a semi-analytic survey. Section 4.3 covers the application of the dual equivalent linearization procedure with the selected weight parameter to analyze the frequency of the nonlinear free oscillations, and some typical cases are then investigated. The approximate solutions proposed by DEL are compared with the exact solutions as well as of some other approximation analytical methods to verify the accuracy and the effect of non-linearity on the effectiveness of the method. offer. The conclusion of Chapter 4 is given in Section 4.4.

4.2. Dual linearization applied to nonlinear deterministic oscillation systems

4.2.1. Weighted dual criterion

4.2.2. Selecting the weight parameter for the predefined dynamic system

$$\frac{(1-p)^n}{1^n}+\frac{p^n}{R^n}=1, n, R>0 \tag{4.17}$$

$$(1-p)^n=\frac{r^2}{1+r^2}, \frac{p^n}{R^n}=\frac{1}{1+r^2} \tag{4.18}$$

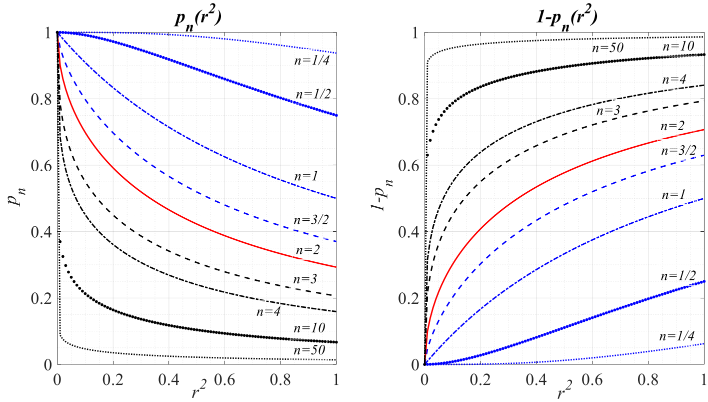


Figure 4.1. Graph of their curve p_n and $1-p_n$ with r^2

For the case n=2 we have

$$p_2=1-\sqrt{\frac{r^2}{1+r^2}} \tag{4.22}$$

The corresponding equivalent linearization coefficient will be

$$k_d=\frac{r}{(1-r^2)\sqrt{1+r^2}+r^3}\frac{\langle AB\rangle}{\langle B^2\rangle} \tag{4.23}$$

4.3. Applying dual linearization in nonlinear free vibration analysis

4.3.1. Dual Linearization Process

Frequency approx

$$\omega_{app}=\sqrt{k_d} \tag{4.30}$$

4.3.2. Problem 1: Oscillator with fraction order restoring force

Table 4.1. Error of approximate frequencies

n	ω_e	ω_c	(%)	ω_{as}	(%)	ω_{aw}	(%)	ω_{d2}	(%)	r^2
1,0	1,000	1,000	0,00	1,000	0,00	1,000	0,00	1,000	0,00	1,000
1,5	0,955	0,957	0,21	0,965	1,05	0,952	-0,25	0,954	-0,06	0,987
2,0	0,915	0,921	0,73	0,931	1,74	0,912	-0,25	0,914	-0,11	0,961
2,5	0,879	0,892	1,42	0,898	2,12	0,878	-0,09	0,878	-0,10	0,930
3,0	0,847	0,866	2,22	0,866	2,22	0,849	0,16	0,847	-0,02	0,900
3,5	0,818	0,844	3,09	0,835	2,07	0,822	0,46	0,819	0,12	0,871
4,0	0,792	0,824	3,99	0,806	1,71	0,799	0,80	0,795	0,30	0,843
4,5	0,769	0,806	4,92	0,777	1,15	0,778	1,16	0,773	0,51	0,818
5,0	0,747	0,791	5,86	0,750	0,42	0,758	1,53	0,752	0,75	0,794
5,5	0,727	0,776	6,79	0,724	-0,45	0,741	1,91	0,734	1,01	0,771
6,0	0,708	0,763	7,72	0,698	-1,45	0,724	2,28	0,717	1,28	0,751
6,5	0,691	0,751	8,65	0,673	-2,56	0,709	2,66	0,702	1,56	0,732
7,0	0,675	0,740	9,56	0,650	-3,77	0,695	3,02	0,687	1,85	0,714
7,5	0,660	0,729	10,46	0,627	-5,06	0,682	3,39	0,674	2,14	0,697
8,0	0,646	0,719	11,35	0,604	-6,43	0,670	3,74	0,662	2,43	0,682
8,5	0,633	0,710	12,22	0,583	-7,86	0,659	4,09	0,650	2,72	0,667
9,0	0,620	0,702	13,08	0,563	-9,33	0,648	4,42	0,639	3,01	0,653
9,5	0,609	0,693	13,93	0,543	-10,85	0,638	4,76	0,629	3,31	0,640
10,0	0,598	0,686	14,76	0,523	-12,41	0,628	5,08	0,619	3,59	0,628

4.3.3. Problem 2: Duffing-harmonic oscillator

Table 4.2. Error of approximate frequencies

a	ω_e	ω_c	(%)	$\omega_{hb}, \omega_{PEM}$	(%)	$\omega_{aw}=\omega_{as}$	(%)	ω_{d2}	(%)	r^2
0,01	0,0093	0,0095	2,22	0,0095	2,22	0,0093	0,15	0,0093	-0,02	0,900
0,05	0,0423	0,0433	2,21	0,0433	2,23	0,0424	0,16	0,0423	-0,02	0,900
0,1	0,0844	0,0862	2,20	0,0863	2,24	0,0845	0,19	0,0844	-0,02	0,901
0,5	0,3874	0,3942	1,77	0,3974	2,58	0,3906	0,83	0,3870	-0,09	0,916
1,0	0,6368	0,6436	1,07	0,6547	2,81	0,6470	1,60	0,6357	-0,17	0,943
2,0	0,8476	0,8507	0,36	0,8660	2,17	0,8615	1,64	0,8463	-0,16	0,976
3,0	0,9196	0,9209	0,14	0,9333	1,49	0,9308	1,21	0,9186	-0,10	0,988
4,0	0,9509	0,9515	0,07	0,9608	1,04	0,9592	0,88	0,9502	-0,07	0,994
5,0	0,9670	0,9673	0,03	0,9744	0,76	0,9733	0,66	0,9665	-0,05	0,996
6,0	0,9763	0,9765	0,02	0,9820	0,58	0,9813	0,51	0,9760	-0,03	0,997
7,0	0,9822	0,9823	0,01	0,9867	0,45	0,9861	0,40	0,9820	-0,02	0,998
8,0	0,9861	0,9862	0,01	0,9897	0,37	0,9893	0,32	0,9860	-0,02	0,999
9,0	0,9889	0,9890	0,01	0,9919	0,30	0,9915	0,27	0,9888	-0,01	0,999
10,0	0,9909	0,9910	0,00	0,9934	0,25	0,9931	0,22	0,9908	-0,01	0,999

4.3.4. Problem 3: Finite extensibility nonlinear oscillator

Table 4.3. Error of approximate frequencies

a	ω_e	ω_c	(%)	ω_{as}	(%)	ω_{lbh}	(%)	ω_{d2}	(%)	r^2
0,100	1,004	1,004	0,00	1,004	0,00	1,004	0,00	1,004	0,00	1,000
0,200	1,015	1,015	0,00	1,015	0,00	1,015	0,00	1,015	0,00	1,000
0,300	1,036	1,036	0,01	1,036	-0,02	1,036	0,00	1,036	0,00	0,999
0,400	1,067	1,067	0,04	1,066	-0,06	1,067	0,00	1,067	0,00	0,998
0,500	1,111	1,112	0,10	1,109	-0,18	1,111	0,00	1,111	-0,01	0,995
0,600	1,176	1,179	0,25	1,170	-0,44	1,175	-0,01	1,175	-0,01	0,988
0,700	1,271	1,278	0,59	1,257	-1,05	1,270	-0,04	1,271	0,00	0,972
0,800	1,423	1,443	1,42	1,387	-2,56	1,420	-0,21	1,424	0,06	0,937
0,825	1,477	1,504	1,80	1,429	-3,24	1,472	-0,32	1,479	0,11	0,923
0,850	1,541	1,577	2,32	1,477	-4,14	1,533	-0,50	1,544	0,17	0,904
0,875	1,619	1,668	3,03	1,533	-5,36	1,606	-0,80	1,624	0,27	0,879
0,900	1,718	1,788	4,06	1,596	-7,08	1,695	-1,31	1,725	0,43	0,846
0,950	2,036	2,209	8,54	1,759	-13,58	1,950	-4,18	2,061	1,26	0,725
0,990	2,769	3,525	27,30	1,943	-29,83	2,305	-16,75	2,913	5,22	0,433

4.3.5. Problem 4: Duffing-type oscillator

Table 4.4. Errors of approximate frequencies, n=1

a	γ	ω_e	ω_c	(%)	ω_{as}	(%)	ω_{aw}	(%)	ω_{d2}	(%)	r^2
0,1	0,1	1,000	1,000	0,00	1,000	0,00	1,000	0,00	1,000	0,00	0,9
1	0,1	1,037	1,037	0,01	1,037	0,01	1,035	-0,13	1,035	-0,14	0,9
10	0,1	2,867	2,915	1,70	2,915	1,70	2,864	-0,11	2,859	-0,26	0,9
100	0,1	26,80	27,400	2,21	27,400	2,21	26,900	0,15	26,800	-0,02	0,9
sai số lớn nhất				2,21		2,21		0,15		-0,26	
a	γ	ω_e	ω_c	(%)	ω_{as}	(%)	ω_{aw}	(%)	ω_{d2}	(%)	r^2
0,1	1,0	1,004	1,004	0,00	1,004	0,00	1,004	-0,01	1,004	-0,02	0,9
1	1,0	1,318	1,323	0,39	1,323	0,39	1,311	-0,48	1,311	-0,55	0,9
10	1,0	8,534	8,718	2,16	8,718	2,16	8,544	0,12	8,529	-0,05	0,9
100	1,0	84,70	86,60	2,22	86,600	2,22	84,900	0,15	84,700	-0,02	0,9
sai số lớn nhất				2,22		2,22		-0,48		-0,55	
a	γ	ω_e	ω_c	(%)	ω_{as}	(%)	ω_{aw}	(%)	ω_{d2}	(%)	r^2
0,1	10	1,037	1,037	0,01	1,037	0,01	1,035	-0,13	1,035	-0,14	0,9
1	10	2,867	2,915	1,70	2,915	1,70	2,864	-0,11	2,859	-0,26	0,9
10	10	26,811	27,404	2,21	27,404	2,21	26,851	0,15	26,805	-0,02	0,9
100	10	267,90	273,90	2,22	273,90	2,22	268,30	0,16	267,90	-0,02	0,9
sai số lớn nhất				2,22		2,22		0,16		-0,26	
a	γ	ω_e	ω_c	(%)	ω_{as}	(%)	ω_{aw}	(%)	ω_{d2}	(%)	r^2

0,1	100	1,318	1,323	0,39	1,323	0,39	1,311	-0,48	1,311	-0,55	0,9
1	100	8,534	8,718	2,16	8,718	2,16	8,544	0,12	8,529	-0,05	0,9
10	100	84,727	86,608	2,22	86,608	2,22	84,859	0,15	84,711	-0,02	0,9
100	100	847,20	866,00	2,22	866,00	2,22	848,50	0,16	847,10	-0,02	0,9
sai số lớn nhất				2,22		2,22		-0,48		-0,55	

CONCLUSIONS AND RECOMMENDATIONS

I. Conclustions:

1. In 3.2 of chapter 3, the author has applied the weighted dual linearization criterion of equivalent linearization to the Euler stability problems. It can be seen that the proposed technique can give more accurate approximations than those obtained by the corresponding Galerkin method. The essence of this method is to determine the equivalence criterion by which to find the linearization coefficients for a given nonlinear system. The accuracy of the linearization coefficients can be improved by using a dual method that combines forward and backward substitution in the weighted duality criterion. The introduction of the weighted parameter p makes the weighted dual mean square error criterion more flexible than the unweighted dual and normal error standard ($p = 1/2$). This result is published in paper 2 as a further development of paper 1 for the case of a column with a fixed end and a hinge end. Through experimental calculation with 7 test functions, it shows that the weight values calculated by formula (3.42) all give solutions with corresponding errors smaller than the error of solutions obtained by Galerkin method. With the problem considered, the weight value p_1 is the value to find. This calculation result opens the way to apply weighted duality criteria to other stability problems.

2. Orthogonality through the mean plays an important role in many fields of science and engineering especially for the Galerkin method. The development of orthogonality through different averaging leads to different approximations. That implies an open question about which appropriate averaging should be used for a given problem to give the most accurate solution. In chapter 3, the author has developed orthogonality to replace the normal averaging (CA) method by weighted local averaging

(WLA) at a local value r introduced. in this study with a weight function of $r^{f(r)}$.

4. A first-order single-parameter weighted polynomial class, $r^{1+\alpha-2\alpha r}$, presented and analyzed in detail. A remarkable feature of these weight functions is that the corresponding WLA coincides with CA at 3 points, that is, $r = 0, r = 0{,}5$ and $r = 1$.

5. The application of the proposed weighted averaging method to the Galerkin method resulted in a GWLA. The global-local approach is implemented to solve the problem of choosing a specific weight function in a class of single-parameter weight functions. The approximations leading to the SGWLA were performed to improve the computation speed while maintaining the accuracy of the solutions obtained by the GWLA.

6. To solve the problem of stability with columns with variable cross-section, the author proposed WLA to apply the method of least squares and convert columns with variable cross-section into equivalent columns with constant cross-section. This result opens up a research direction that can be applied in many practical engineering problems.

7. The application of GWLA and SGWLA to determining the critical load of columns with different boundary and cross-sectional conditions is presented in chapter 3. Numerical calculations show that GWLA and SGWLA offer significant improvements about the approximate critical load accuracy relative to the results obtained by the GCA for the columns under consideration.

The results in chapter 3 are published in publication number [1], [2], [5].

8. Developing the orthogonal property through which some basic properties of dual equivalence linearization are determined, showing the similarities and differences between one-way and two-way substitution processes. Accordingly, it also shows that the weights are closely related to the squared correlation coefficient.

9. Based on the dual approach, the forward and backward substitution processes can be considered in two forms: The first form is the one-way substitution processes, which are represented by two single-objective optimization problems. The second is the two-way substitution process represented by the weighted sum of the two single objective functions mentioned above. In all these substitution processes, the squared correlation coefficient, which is a measure of the degree of nonlinearity between the original nonlinear term and the equivalent linear term, occurs naturally in the expression of the linear coefficient. equivalence calculation and optimal substitution error.

10. A family of weighting parameters is proposed based on a semi-analytic analysis of the contribution of the optimal first and second substitution errors to their sum. Furthermore, a weight described by the equation of an ellipse is chosen from this family.

11. The dual linearization procedure with the selected weight parameter is built for a class of nonlinear deterministic oscillators. Applied to some typical systems, it shows that the selected weight parameter is a suitable choice to analyze the free oscillation frequency of the considered nonlinear deterministic systems. Besides, the evaluation of the error level based on the squared correlation coefficient shows a clear influence of nonlinearity on the current approximations.

12. It can be seen that the proposed method has a great potential and it needs to be explored for broader classes of nonlinear deterministic dynamic systems. In particular, the proposed weight parameter selection can be extended and further studied for other problems of multi-objective optimization..

13. The results of developing orthogonal properties through the proposed dual linearization method give the lowest absolute maximum error among the approximate frequencies obtained from some other analytical methods when considering with relatively large degrees of nonlinearity. Its absolute maximum error is only about 5%, which is acceptable in the preliminary engineering design.

The research results of chapter 4 are published in publication number [6].

II. Recomendations

1. The results of developing orthogonality and providing a suitable weighted parameter have opened up the research direction to apply the weighted dual standard to other stability problems.

2. The results of developing the orthogonality through the proposed WLA can be verified for columns with other boundary conditions, more complex variable cross-sections and also extended to the transformation problems of the columns. more complex structures such as columns, plates, and nonlinear shells where the use of the analytic form of the Galerkin method is often limited to first-order approximations.

3. With WLA apply the method of least squares and convert the column with variable cross-section into the equivalent column with constant cross-section. This result opens a research direction that can be applied in many engineering problems in practice.

4. The improved DEL method with the selected weight parameter p_2 has a great potential and it needs to be explored for broader classes of nonlinear deterministic dynamics and can be considered to apply to classes of problems of forced nonlinear oscillation systems and other oscillating systems. In particular, the proposed selection of weighting coefficients can be extended and further studied for other problems of multi-objective optimization.

NEW CONTRIBUTIONS OF THE THESIS

1. Developing orthogonality in the approximation through weighted dual linearization combined with the proposal of specific forms of the weighted parameter p gives better error approximations than the Galerkin method apply stability problems while developing the weight parameter more flexible than normal error standard and unweighted duality. (p=1/2)

2. Developing the orthogonality by constructing a new weighted local average (WLA) applied to the Galerkin method, Galerkin proposes that the Galerkin method uses the weighted local average (GWLA) to significantly improve the accuracy. exactness of the first-order approximation of the Galerkin method.

3. Presenting the local - global approach to solve the problem of choosing a single-parameter weight function and proposing the simplified GWLA method (SGWLA) to reduce the computational weight but still maintain the accuracy of the solutions received from the GWLA.

4. Combine WLA with least squares to convert columns of variable cross-section into equivalent columns of constant cross-section, and these new numerical algorithms can provide a new and efficient alternative to computation techniques in the design of structural systems with variable cross-sections.

5. Developing orthogonality through dual linearization and determining the squared correlation coefficient, which is a measure of the degree of nonlinearity between the original nonlinear term and the naturally occurring equivalent linear term in the expression of the equivalent linearization coefficient and the optimal substitution error.

6. Proposing a family of weighting parameters (in the form of an ellipse equation) based on a semi-analytic analysis of the contribution of optimal forward and backward substitution errors to their sum in dynamic system analysis nonlinear predestination.

7. Building a weighted dual linearization process for a class of nonlinear deterministic dynamic systems, choosing a suitable weighted parameter and effectively applying it in the analysis of the free oscillation frequency of a number typical routing money system.

8. The absolute maximum error of the frequency approximation from the proposed method compared with other approximations is about 5%, which is accepted in the preliminary engineering design.

LIST OF PUBLICATIONS

1. Trần Tuấn Long, Nguyễn Đông Anh, Nguyễn Xuân Thành, "Weighting dual technique of equivalent linearization method for Euler stability problem," in *Hội nghị Cơ học toàn quốc lần thứ X*, Hà Nội, 2017.

2. Nguyễn Đông Anh, Trần Tuấn Long, Nguyễn Xuân Thành, "Cách tiếp cận đối ngẫu áp dụng cho bài toán ổn định Euler dạng thanh một đầu ngàm, một đầu khớp," in *Hội nghị Cơ học kỹ thuật toàn quốc (Kỷ niệm 40 năm thành lập Viện Cơ học)*, Hà Nội, 09/04/2019.

3. Nguyen, T. X., Tran, L. T. "*A High-Order time finite element method applied to structural dynamics problems*". Proceedings of ICOMMA - In Modern Mechanics and Applications, Springer, pp 137–148, 2020.

4. Nguyen, T. X., & Tran, L. T. *A simplified variant of the time finite element methods based on the shape functions of an axial finite bar. Journal of Science and Technology in Civil Engineering (STCE) - HUCE, 15(4), pp 42-53, 2021. https://doi.org/10.31814/stce.huce(nuce)2021-15(4)-04.*

5. Anh Tay Nguyen, Nguyen Cao Thang, Tran Tuan Long, N. D. Anh, P. M. Thang and Nguyen Xuan Thanh "*A novel weighted local averaging for the Galerkin method with application to elastic buckling of Euler column,*" *International Journal for Computational Methods in Engineering Science and Mechanics, Volume 24, Issue 2, Pages 128-142. May 30, 2022. https://doi.org/10.1080/15502287.2022.2080612.*

6. Anh N. D., Nguyen Ngoc Linh, Tran Tuan Long, Nguyen Cao Thang, Nguyen Tay Anh. I. Elishakoff. "*Extension of dual equivalent linearization to analysis of deterministic dynamic systems*. Part 1: Single-parameter equivalent linearization," *Nonlinear Dynamics, 111, pages 997–1017, Nov, 28th 2022.*

https://doi.org/10.1007/s11071-022-07894-6.