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DARK MATTER AND NEUTRINO MASS IN THE 3 – 4 – 1 – 1 MODEL

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SUMMARY OF PHYSICS DOCTORAL THESIS

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- Institute of Physics Library;
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INTRODUCTION

1. The urgency of the thesis

It can be stated that standard model of particle physics is the most successful in describing strong interaction with $SU(3)_c$ gauge symmetry and unification of electroweak interaction with the $SU(2)_L \times U(1)_Y$ gauge symmetry. Because the predictions of standard model have been empirically tested, for example, the existence of $W^{\pm}, Z$, quark c, t, neutral currents, mass of $W^{\pm}, Z$ ..., especially, predicting the existence of the Higgs boson. However, it leaves a number of striking physics features of our world unexplained.

Since some issues has yet to explain by the standard model, there are a lot of the new physical models suggested and mainly focus on explaining recent data and experiments such as dark matter, neutrino mass, new physics. Therefore, we focus studying a class of models based upon gauge symmetry, $SU(3)_C \times SU(P)_L \times U(1)_X \times U(1)_N$, called 3−P−1 − 1. Indeed, The distinct advantage of this class of models is possible to find a symmetry breaking mechanism such that the residual gauge symmetry plays the role to ensure the stability of dark matter candidate. The version with $P = 3$ has shown a residual gauge symmetry as a group Z_2 well established in the literature, when the dark matter candidates can be one of the fields which Z_2 is odd. In this work, we investigate the class of model for P integer and arbitrary. We prove that the model with $4 \leq P$ can exist more than one residual discrete symmetry group, therefore the model prediction provides the novel scenarios of multicomponent dark matter. Where The minimal of multicomponent dark matter according to $P = 4$, the $3-4-1-1$ model, containing the two component dark matter. Since the theory contains two commutative groups U_1, containing the kinetic term between two U_1 factors. Therefore, the $3 - 4 - 1 - 1$ model has new physical effect which associated with interactions of bosons, called kinetic mixing effect. Beside, as a direct result, the neutrinos obtain appropriate masses via a canonical seesaw mechanism.

For the mentioned reasons, we choose the subject: "Dark matter and neutrino mass in the $3 - 4 - 1 - 1$ model"

2. The objectives of the thesis

- Study the mixing effect of gauge bosons in the $3-4-1-1$ model which take into the kinetic mixing term.
- Solve the problem of the multicomponent dark matter in the $3-4-1-1$ model.

3. The main research contents of the thesis

- Overview of standard model.
- Studying the theory of the $3-4-1-1$ model.
- Investigating the $3-4-1-1$ model with the kinetic mixing effect.
- Investigating the $3-4-1-1$ model with multicomponent dark matter.

CHAPTER 1. OVERVIEW OF STANDARD MODEL

1.1. The standard model

1.1.1. The gauge symmetry of standard model

The standard model describes strong, electromagnetic, and weak interactions based upon gauge symmetry group, $SU(3)_C \times SU(2)_L \times U(1)_Y$, called $(3-2-1)$. Where, the gauge group $SU(3)_C$ describes strong interaction, the gauge group $SU(2)_L \times U(1)_Y$ describes weak electrical interaction.

1.1.2. Particle presentation

In the standard model, the left-handed fermions are $SU(2)_L$ doublets, while the right-handed fermions are $SU(2)_L$ singlets. Beside the electric charge operator $Q = T_3 + Y/2$.

The fermions are leptons and quarks in standard model which are arranged under the gauge symmetry group as follows:

$$\begin{aligned} &\psi_{aL} = \begin{pmatrix} \nu_{aL} \\ e_{aL} \end{pmatrix} \sim (1, 2, -1), \\ &e_{aR} \sim (1, 1, -2) \quad . \end{aligned} \tag{1.1}$$

$$\begin{aligned} &Q_{aL} = \begin{pmatrix} u_{aL} \\ d_{aL} \end{pmatrix} \sim \left(3, 2, \frac{1}{3}\right), \\ &u_{aR} \sim \left(3, 1, \frac{4}{3}\right), \quad d_{aR} \sim \left(3, 1, -\frac{2}{3}\right), \end{aligned} \tag{1.2}$$

when $a = 1, 2, 3$ is the generation index.

Scalar bosons (called Higgs bosons, ϕ) is included into the model and transformed under the gauge symmetry group as :

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \sim (1, 2, 1). \tag{1.3}$$

1.1.3. Lagrangian

The total Lagrangian has the form,

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{Yuk} + V_\phi + \mathcal{L}_{gf} + \mathcal{L}_{FPG}, \tag{1.4}$$

Where the first part combines kinetic terms and gauge interactions, the second and third parts are the Yukawa interaction and the scalar potential, the two last parts are the gauge fixing and the ghost terms, respectively.

1.1.4. The scalar potential and Higgs mechanism

The Higgs pontential $V(\phi)$ as follows:

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \frac{\lambda}{4}(\phi^\dagger \phi)^2. \tag{1.5}$$

Using the potential minimun conditions: $v = \sqrt{-\frac{\mu^2}{\lambda}}$, and $\mu^2 < 0, \lambda > 0$. It is easily check that get the physical Higgs particle h and two massless Goldstone boson, $G_Z, G_{W^\pm}$.

The ϕ field expanded according to the physical state which has the form as:

$$\phi = \begin{pmatrix} G_W^+ \\ \frac{v+h+iG_Z}{\sqrt{2}} \end{pmatrix}. \tag{1.6}$$

The mass Higgs boson h may be written as:

$$m_h^2 = 2\lambda v^2. \tag{1.7}$$

The experiments determined the mass of h:

$$m_h = 125.09 \pm 0.24 GeV. \tag{1.8}$$

1.2. Fermion mass

The Yukawa interaction is given by:

$$\mathcal{L}_{Yuk} = Y_{ij}^e \bar{\psi}_L^i \phi e_R^j + Y_{ij}^d \bar{Q}_L^i \phi d_R^j + Y_{ij}^u \bar{Q}_L^i (i\sigma_2 \phi^*) u_R^j + H.c., \tag{1.9}$$

where $Y_{ij}^{e,d,u}$ is 3×3 matrix, called Yukawa interaction constant.

After the diagonalization of the mass matrices, the physical mass of the charged leptons and quarks are given by:

$$m_e = \frac{y_e v}{\sqrt{2}}, \quad m_\mu = \frac{y_\mu v}{\sqrt{2}}, \quad m_\tau = \frac{y_\tau v}{\sqrt{2}}, \tag{1.10}$$

$$m_u = \frac{y_u v}{\sqrt{2}}, \quad m_c = \frac{y_c v}{\sqrt{2}}, \quad m_t = \frac{y_t v}{\sqrt{2}}, \tag{1.11}$$

$$m_d = \frac{y_d v}{\sqrt{2}}, \quad m_s = \frac{y_s v}{\sqrt{2}}, \quad m_b = \frac{y_b v}{\sqrt{2}}, \tag{1.12}$$

$$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0. \tag{1.13}$$

It is clear that the neutrino in standard model exactly have zero mass.

1.3. Gauge bosons mass - Interaction of gauge bosons

In standard model, the gauge bosons transform under the gauge group as follow:

$$G^\mu_a \sim (8,1,0), W^\mu_a \sim (1,3,0), B^\mu \sim (1,1,0) \quad a = 1,2,3. \tag{1.14}$$

The kinetic terms of the fields in standard model has form as:

$$\begin{aligned} \mathcal{L}_{kin} &= -\frac{1}{4}G^{\mu\nu_a}G_{\mu\nu a} - \frac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &-i\bar{Q}_{aL}\gamma^\mu D_\mu Q_{aL} - i\bar{\psi}_{aL}\gamma^\mu D_\mu \psi_{aL} - i\bar{e}_{aR}\gamma^\mu D_\mu e_{aR} \\ &-i\bar{u}_{aR}\gamma^\mu D_\mu u_{aR} - i\bar{d}_{aR}\gamma^\mu D_\mu d_{aR} - (D^\mu\phi)^+(D^\mu\phi). \end{aligned} \tag{1.15}$$

The mass spectrum of gauge bosons are determined the covariant derivative, given by:

$$\mathcal{L}_{gauge-mass} = (D^\mu < \phi >)^+(D^\mu < \phi >), \tag{1.16}$$

where:

$$D_\mu < \phi > = (\partial_\mu + igT_a W^a_\mu + ig' B_\mu) < \phi > . \tag{1.17}$$

With the physical basic, the equation (1.16) is written:

$$\mathcal{L}_{gauge-mass} = \frac{1}{4}g^2 v^2 W^{+\mu}W^-_\mu + \frac{1}{8}(g^2 + g'^2)v^2 Z^\mu Z_\mu. \tag{1.18}$$

By the equation (1.18), the gauge boson masses $A, Z, W^\pm$ is:

$$m_A = 0, \qquad m_Z = \frac{gv}{2c_W}, \qquad m_{W^\pm} = \frac{gv}{2}. \tag{1.19}$$

Standard model predicts $\rho = \frac{m^2_W}{m^2_Z c^2_{\theta_W}} = 1$. However, the experiment determined: $m_W = 80.385 \pm 0.015$ GeV, $m_Z = 91.1876 \pm 0.0021$ GeV. Therefore, there is a deviation in the ρ parameter determined by standard model and experiment. Thus, there may be a new physics contribution to the parameter ρ.

1.4. The neutral current

Lagrangian in equation (1.15) contains the interactions of the neutral bosons with fermion. Where, the neutral current has form as:

$$\mathcal{L}_{NC} = g s_W J_\mu^{em} A^\mu + \frac{g}{2c_W} J_\mu^0 Z^\mu, \tag{1.20}$$

where J_μ^{em}, J_μ^0 are the charged current and the neutral current, respectively:

$$\begin{aligned} J_\mu^{em} &= Q(f)\bar{f}\gamma_\mu f, \\ J_\mu^0 &= \bar{f}\gamma_\mu \left[g_V^{Z_\mu}(f) - g_A^{Z_\mu}(f)\gamma_5 \right] f, \end{aligned} \tag{1.21}$$

constant g_V^f and g_A^f are given by:

$$g_V^f = g_L^f + g_R^f \qquad\qquad g_A^f = g_L^f - g_R^f, \tag{1.22}$$

From the particle structure and the particles interactions in standard model, we find that the standard model does not contain candidates which satisfy the characters of dark matter.

1.5. Conclusion

In this chapter, we present some highlight characters of standard model. Beside, we also presents a few unresolved problems within the standard model which relate to the content of thesis as neutrino mass, dark matter, parameter ρ.

CHAPTER 2. MULTICOMPONENT DARK MATTER IN NONCOMMUTATIVE $B-L$ GAUGE THEORY

2.1. Noncommutative $(B-L)$ gauge theory

While symmetry group describes strong interactions which are preserved, the $SU(2)_L$ symmetry of weak isospin be enlarged to $SU(P)_L$, for $P = 3, 4, 5, \cdots$. a higher weak issospin symmetric.

- The left-handed leptons are P-plet of $SU(P)_L$:

$$\psi_{aL} = \begin{pmatrix} \nu^{0,-1} \\ e^{-1,-1} \\ E_1^{q_1,n_1} \\ E_2^{q_2,n_2} \\ \vdots \\ E_{P-2}^{q_{P-2},n_{P-2}} \end{pmatrix}_{aL} \sim P, \tag{2.1}$$

The left-handed quarks of the first and second generation are $SU(P)_L$ anti-P-plet, while their the third generation are $SU(P)_L$ P-plet are given by:

$$Q_{\alpha L} = \begin{pmatrix} d^{-1/3,1/3} \\ -u^{2/3,1/3} \\ J_1^{-q_1-1/3,-n_1-2/3} \\ J_2^{-q_2-1/3,-n_2-2/3} \\ \vdots \\ J_{P-2}^{-q_{P-2}-1/3,-n_{P-2}-2/3} \end{pmatrix}_{\alpha L} \sim P^*, \tag{2.2}$$

$$Q_{3L} = \begin{pmatrix} u^{2/3,1/3} \\ d^{-1/3,1/3} \\ J_1^{q_1+2/3,n_1+4/3} \\ J_2^{q_2+2/3,n_2+4/3} \\ \vdots \\ J_{P-2}^{q_{P-2}+2/3,n_{P-2}+4/3} \end{pmatrix}_{3L} \sim P, \tag{2.3}$$

- The right-handed fermions trasform as $SU(P)_L$ singlets:

$$\nu_{aR} \sim (1), e_{aR} \sim (1), E_{kaR} \sim (1) \tag{2.4}$$
$$u_{aR} \sim (1), d_{aR} \sim (1), J_{k\alpha R} \sim (1) \tag{2.5}$$
$$E_{k3R} \sim (1), J_{k3R} \sim (1). \tag{2.6}$$

Both electric charge Q and Baryon minus Lepton charge $(B-L)$ neither commute nor close algebraically with $SU(P)_L$. Nontrivial communitation relations are obtained by::

$$[Q, T_1 \pm iT_2] = \pm(T_1 \pm iT_2), \tag{2.7}$$
$$[Q, T_4 \pm iT_5] = \mp q_1(T_4 \pm iT_5), \tag{2.8}$$
$$[Q, T_6 \pm iT_7] = \mp(1+q_1)(T_6 \pm iT_7), \tag{2.9}$$
$$[Q, T_9 \pm iT_{10}] = \mp q_2(T_9 \pm iT_{10}), \tag{2.10}$$
$$[Q, T_{11} \pm iT_{12}] = \mp(1+q_2)(T_{11} \pm iT_{12}), \tag{2.11}$$
$$[Q, T_{13} \pm iT_{14}] = \mp(q_2 - q_1)(T_{13} \pm iT_{14}), \tag{2.12}$$
$$\cdots\cdots\cdots,$$
$$[Q, T_{P^2-3} \pm iT_{P^2-2}] = \mp(q_{P-2} - q_{P-3})(T_{P^2-3} \pm iT_{P^2-2}), \tag{2.13}$$

$$[B-L, T_4 \pm iT_5] = \mp(1+n_1)(T_4 \pm iT_5), \tag{2.14}$$
$$[B-L, T_6 \pm iT_7] = \mp(1+n_1)(T_6 \pm iT_7), \tag{2.15}$$
$$[B-L, T_9 \pm iT_{10}] = \mp(1+n_2)(T_9 \pm iT_{10}), \tag{2.16}$$
$$[B-L, T_{11} \pm iT_{12}] = \mp(1+n_2)(T_{11} \pm iT_{12}), \tag{2.17}$$
$$[B-L, T_{13} \pm iT_{14}] = \mp(n_2 - n_1)(T_{13} \pm iT_{14}), \tag{2.18}$$
$$\cdots\cdots\cdots,$$
$$[B-L, T_{P^2-3} \pm iT_{P^2-2}] = \mp(n_{P-2} - n_{P-3})(T_{P^2-3} \pm iT_{P^2-2}), \tag{2.19}$$

To close the symmetry, two Abelian charges must be imposed, yielding a complete gauge symmetry. The gauge symmetry is given as:

$$SU(3)_C \times SU(P)_L \times U(1)_X \times U(1)_N, \tag{2.20}$$

called 3−P−1 − 1, where the color group is also included $SU(3)_C$, and X, N determines Q and $(B-L)$, respectively :

$$Q = \sum_{k=0}^{P-2} \beta_k H_k + X, \qquad B-L = \sum_{k=0}^{P-2} b_k H_k + N, \tag{2.21}$$

Here $H_k = T_{(k+2)^2-1} = T_3, T_8, T_{15}, \cdots, T_{P^2-1}$ according to $k = 0, 1, 2, \cdots, P-2$ are the $SU(P)_L$ Cartan generators.

To summarize the full fermion content transforms under the 3−P−1−1 symmetry as:

$$\psi_{aL} \sim \left(1, P, \frac{q-1}{P}, \frac{n-2}{P}\right), \tag{2.22}$$

$$Q_{\alpha L} \sim \left(3, P^*, -\frac{1}{3} + \frac{1-q}{P}, -\frac{2}{3} + \frac{2-n}{P}\right), \tag{2.23}$$

$$Q_{3L} \sim \left(3, P, \frac{2}{3} + \frac{q-1}{P}, \frac{4}{3} + \frac{n-2}{P}\right), \tag{2.24}$$

$$\nu_{aR} \sim (1,1,0,-1), e_{aR} \sim (1,1,-1,-1), E_{kaR} \sim (1,1,q_k,n_k), \tag{2.25}$$

$$u_{aR} \sim (3,1,2/3,1/3), d_{aR} \sim (3,1,-1/3,1/3), \tag{2.26}$$

$$J_{k\alpha R} \sim (3,1,-q_k-1/3,-n_k-2/3),$$

$$J_{k3R} \sim (3,1,q_k+2/3,n_k+4/3), \tag{2.27}$$

where we denote $k = 1, 2, 3, \cdots, P-2$, $q \equiv q_1 + q_2 + \cdots + q_{P-2}$ and $n \equiv n_1 + n_2 + \cdots + n_{P-2}$.

To break the gauge symmetry and generate the particle masses properly, we introduce P scalar P-plets plus a scalar singlet:

$$\begin{pmatrix} \varphi_{11}^{0,0} \\ \varphi_{21}^{-1,0} \\ \varphi_{31}^{q_1,n_1+1} \\ \varphi_{41}^{q_2,n_2+1} \\ \vdots \\ \varphi_{P1}^{q_{P-2},n_{P-2}+1} \end{pmatrix}, \quad \begin{pmatrix} \varphi_{12}^{1,0} \\ \varphi_{22}^{0,0} \\ \varphi_{32}^{q_1+1,n_1+1} \\ \varphi_{42}^{q_2+1,n_2+1} \\ \vdots \\ \varphi_{P2}^{q_{P-2}+1,n_{P-2}+1} \end{pmatrix},$$

$$\begin{pmatrix} \varphi_{13}^{-q_1,-1-n_1} \\ \varphi_{23}^{-1-q_1,-1-n_1} \\ \varphi_{33}^{0,0} \\ \varphi_{43}^{q_2-q_1,n_2-n_1} \\ \vdots \\ \varphi_{P3}^{q_{P-2}-q_1,n_{P-2}-n_1} \end{pmatrix}, \tag{2.28}$$

$$\begin{pmatrix} \varphi_{14}^{-q_2,-1-n_2} \\ \varphi_{24}^{-1-q_2,-1-n_2} \\ \varphi_{34}^{q_1-q_2,n_1-n_2} \\ \varphi_{44}^{0,0} \\ \vdots \\ \varphi_{P4}^{q_{P-2}-q_2,n_{P-2}-n_2} \end{pmatrix}, \cdots\cdots\cdots, \begin{pmatrix} \varphi_{1P}^{-q_{P-2},-1-n_{P-2}} \\ \varphi_{2P}^{-1-q_{P-2},-1-n_{P-2}} \\ \varphi_{3P}^{q_1-q_{P-2},n_1-n_{P-2}} \\ \varphi_{4P}^{q_2-q_{P-2},n_2-n_{P-2}} \\ \vdots \\ \varphi_{PP}^{0,0} \end{pmatrix},$$

$$\phi \sim (1,1,0,2). \tag{2.29}$$

To be consistent with standard model, we impose $v_{1,2} \ll v_{3,4,5,\cdots,P}, \Lambda$. The scheme of the gauge symmetry breaking is therefore summarized as:

$$\begin{array}{c} SU(3)_C \times SU(P)_L \times U(1)_X \times U(1)_N \\ \downarrow v_{3,4,\cdots,P}, \Lambda \\ SU(3)_C \times SU(2)_L \times U(1)_Y \times P \\ \downarrow v_{1,2} \\ SU(3)_C \times U(1)_Q \times P \end{array}$$

While P is a residual symmetry of $(B-L)$. We further redefine:

$$P = (-1)^{3(B-L)+2s}, \tag{2.30}$$

2.2. The minimal multicomponent dark matter model

The gauge symmetry is given by:

$$SU(3)_C \times SU(4)_L \times U(1)_X \times U(1)_N. \tag{2.31}$$

The Q and $(B-L)$ charges are embedded:

$$Q = T_3 + \beta T_8 + \gamma T_{15} + X, \qquad B - L = bT_8 + cT_{15} + N, \tag{2.32}$$

The VEVs of scalar multiplets are written as:

$$\langle\eta\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \langle\rho\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \\ 0 \\ 0 \end{pmatrix}, \tag{2.33}$$

$$\langle\chi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ w \\ 0 \end{pmatrix}, \qquad \langle\Xi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 0 \\ V \end{pmatrix}, \qquad \langle\phi\rangle = \frac{1}{\sqrt{2}}\Lambda, \tag{2.34}$$

The multiple matter parity takes the form:

$$P = P_n \otimes P_m, \tag{2.35}$$

where the partial parities P_n and P_m have values to be either 1 or $P_n^{\pm} = (-1)^{\pm(3n+1)} = -1$ và $P_m^{\pm} = (-1)^{\pm(3m+1)} = -1$, provided that $n, m = 2z/3 = 0, \pm 2/3, \pm 4/3, \pm 2, \cdots$ respectively. P classifies the particles, such as:

1. Normal particles for $P = (+, +)$, which include the standard model particles.
2. Wrong particles for $P = (+, -)$, $(-, +)$ or $(-, -)$, containing the most new particles.

The dark matter candidate can be viable to be a new leptons, the physical scalar and gauge boson fileds.

Model	$(-,+)$ Cadidate	$(+,-)$ Cadidate	$(-,-)$ Cadidate
$q = p = 0$	$E_{1,2,3}$, $\mathcal{H}_2$, W_{13}	$F_{1,2,3}$, $\mathcal{H}_3$, W_{14}	$\mathcal{H}_6$, W_{34}
$q = 0, p = -1$	$E_{1,2,3}$, $\mathcal{H}_2$, W_{13}	$\mathcal{H}_5$, W_{24}	Non
$q = -1, p = 0$	$\mathcal{H}_4$, W_{23}	$F_{1,2,3}$, $\mathcal{H}_3$, W_{14}	Non
$q = p = -1$	$\mathcal{H}_4$, W_{23}	$\mathcal{H}_5$, W_{24}	$\mathcal{H}_6$, W_{34}
$q = p \neq 0, -1$	Non	Non	$\mathcal{H}_6$, W_{34}

Table 2.4: The dark matter candidates of different versions of the $3-4-1-1$ model.

In the results listed in the table 2.4, we found that the version with $p = q = 0$ has a rich two component dark matter structure, to be further investigate next section.

2.3. Studying dark matter in the $3-4-1-1$ models with $p = q = 0$

We investigate the two component dark matter version to $q = p = 0$. The total Lagrangian under gauge symmetry (up to the gauge fixing and ghost terms) is generally defined as follow:

$$\mathcal{L} = \bar{F} i\gamma^\mu D_\mu F + (D^\mu S)^\dagger (D_\mu S) - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} + \mathcal{L}_{\text{Yukawa}} - V_{\text{Higgs}}, \quad (2.36)$$

where F, S and A run over the fermion, scalar, and gauge boson multiplets.

The Yukawa interactions can be extracted from Appendix B.

The scalar potential is given by:

$$\begin{aligned} V_{\text{Higgs}} &= \mu_1^2 \eta^\dagger \eta + \mu_2^2 \rho^\dagger \rho + \mu_3^2 \chi^\dagger \chi + \mu_4^2 \Xi^\dagger \Xi + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\rho^\dagger \rho)^2 \\ &+ \lambda_3 (\chi^\dagger \chi)^2 + \lambda_4 (\Xi^\dagger \Xi)^2 + (\eta^\dagger \eta)(\lambda_5 \rho^\dagger \rho + \lambda_6 \chi^\dagger \chi + \lambda_7 \Xi^\dagger \Xi) \\ &+ (\rho^\dagger \rho)(\lambda_8 \chi^\dagger \chi + \lambda_9 \Xi^\dagger \Xi) + \lambda_{10} (\chi^\dagger \chi)(\Xi^\dagger \Xi) + \lambda_{11} (\eta^\dagger \rho)(\rho^\dagger \eta) \end{aligned}$$

$$
\begin{aligned}
&+\lambda_{12}(\eta^\dagger\chi)(\chi^\dagger\eta)+\lambda_{13}(\eta^\dagger\Xi)(\Xi^\dagger\eta)+\lambda_{14}(\rho^\dagger\chi)(\chi^\dagger\rho)\\
&+\lambda_{15}(\rho^\dagger\Xi)(\Xi^\dagger\rho)+\lambda_{16}(\chi^\dagger\Xi)(\Xi^\dagger\chi)\\
&+(\lambda_{17}\eta\rho\chi\Xi+H.c.)+V(\phi),
\end{aligned} \tag{2.37}
$$

where the last term is the potential of ϕ plus the interactions of ϕ with η, ρ, χ and Ξ:

$$
V(\phi)=\mu^2\phi^*\phi+\lambda(\phi^*\phi)^2+(\phi^*\phi)(\lambda_{18}\eta^\dagger\eta+\lambda_{19}\rho^\dagger\rho+\lambda_{20}\chi^\dagger\chi+\lambda_{21}\Xi^\dagger\Xi). \tag{2.38}
$$

2.3.1. Scalar and gauge sectors

In summary, the physical neutral gauge bosons are related to the beginning states by $(A\ Z_1\ Z_2\ Z_3)^T=U(A_3\ A_8\ A_{15}\ B)^T$. Where, The basic transfer matrix U is given :

$$
U=\begin{pmatrix}
s_W & \beta s_W & \gamma s_W & \frac{s_W}{t_X}\\
c_W & -\beta s_W t_W & -\gamma s_W t_W & -\frac{s_W t_W}{t_X}\\
0 & c_\varphi\sqrt{1-\beta^2t_W^2} & -\frac{s_\varphi}{\sqrt{1+\gamma^2t_X^2}}-\frac{c_\varphi\beta\gamma t_W^2}{\sqrt{1-\beta^2t_W^2}} & \frac{s_\varphi\gamma t_X}{\sqrt{1+\gamma^2t_X^2}}-\frac{c_\varphi\beta t_W^2}{t_X\sqrt{1-\beta^2t_W^2}}\\
0 & s_\varphi\sqrt{1-\beta^2t_W^2} & \frac{c_\varphi}{\sqrt{1+\gamma^2t_X^2}}-\frac{s_\varphi\beta\gamma t_W^2}{\sqrt{1-\beta^2t_W^2}} & -\frac{c_\varphi\gamma t_X}{\sqrt{1+\gamma^2t_X^2}}-\frac{s_\varphi\beta t_W^2}{t_X\sqrt{1-\beta^2t_W^2}}
\end{pmatrix}.
$$

2.3.2. Interactions

Gauge interactions for fermions

In Appendix C, we compute the couplings of Z_1 with fermions

Gauge interactions for scalars

In Appendix D, we calculate all the gauge boson and scalar interactions.

2.4. Multicomponent dark matter phenomenology

We consider the model with $q=p=0$. In this case, the neutral particles that transform nontrivially under the multiple matter parity $P=P_n\otimes P_m$ are E_a^0, F_a^0, $\mathcal{H}_2^0$, $\mathcal{H}_3^0$, $\mathcal{H}_6^0$, W_{13}^0, W_{14}^0 , and W_{34}^0, as explicitly shown in table 2.4. We divide into three possibilities of two component dark matter existence.

2.4.1. Scenario with two fermion dark matter

We assume that E (one of three particles E_a^0) and F (one of three particles F_a^0), which are singly-wrong particles according to the separately conserved single parities P_n and P_m, are the lightest particles within the classes of singly-wrong particles of the same kind ($E_a, \mathcal{H}_2$, W_{13}), and ($F_a, \mathcal{H}_3$, W_{14}), respectively.

The dominant channels of the dark matter pair annihilation into the standard model particles are give by:

$$EE^c \rightarrow \nu\nu^c, l^- l^+, qq^c, Z_1 H_1, \quad (2.39)$$

$$FF^c \rightarrow \nu\nu^c, l^- l^+, qq^c, Z_1 H_1, \quad (2.40)$$

There is the conversion between dark matter scenario, in which the heavier dark matter component would annihilate into the lighter one. In this sense, there adds the annihilation process either:

$$EE^c \rightarrow FF^c \quad \text{if} \;\; m_E > m_F, \quad (2.41)$$

$$FF^c \rightarrow EE^c \quad \text{if} \;\; m_F > m_E. \quad (2.42)$$

The dark matter pair annihilation into the standard model particles and conversion between dark matter components are given figures 2.1 and 2.2, respectively:

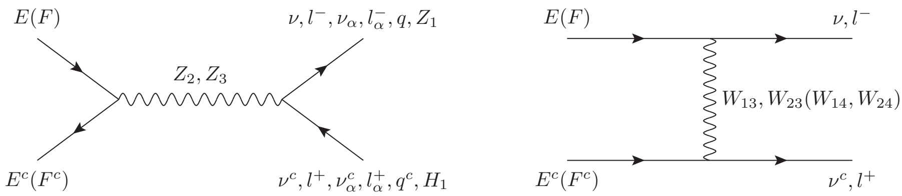

Fig 2.1: Dominant contributions to annihilation of the two component fermion dark matter into standard model particles.

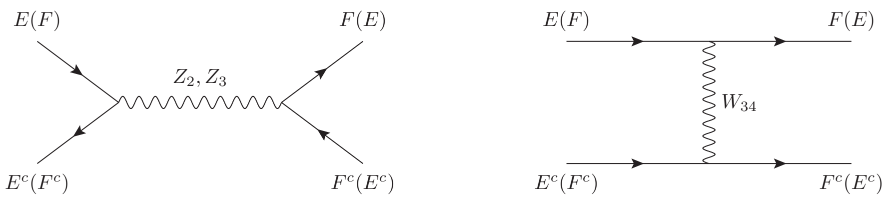

Fig 2.2: Conversion between fermion dark matter components.

One can obtain the individual relic abundance of each dark matter component as:

$$\Omega_E h^2 = 2.752 \frac{m_E}{\text{GeV}} Y_E(x_\infty) \times 10^8, \quad (2.43)$$

$$\Omega_F h^2 = 2.752 \frac{m_F}{\text{GeV}} Y_F(x_\infty) \times 10^8, \quad (2.44)$$

The dark matter relic abundance is a sum of the individual contributions as:

$$\Omega_{\text{DM}} h^2 = \Omega_E h^2 + \Omega_F h^2. \quad (2.45)$$

For numerical investigation, we use the following parameter values throughout this thesis: $u = v \simeq 174$ GeV, $s_W^2 \simeq 0.231$, $g = \sqrt{4\pi\alpha}/s_W$, $m_{Z_1} = 91.187$ GeV. Additionally, the atomic numbers of Xenon are: $Z = 54$ and $A = 131$.

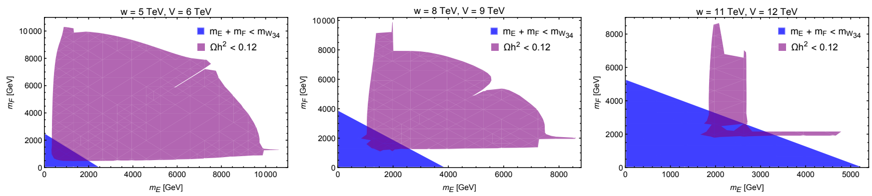

Fig 2.3: The total relic density contoured as a function of (m_E, m_F), where the dark matter stable regime is also included, according to the several choices of w, V.

In figure 2.3, we show the viable dark matter mass regime as the overlap of the two colored regions according to the relic density and the stability condition, respecively.

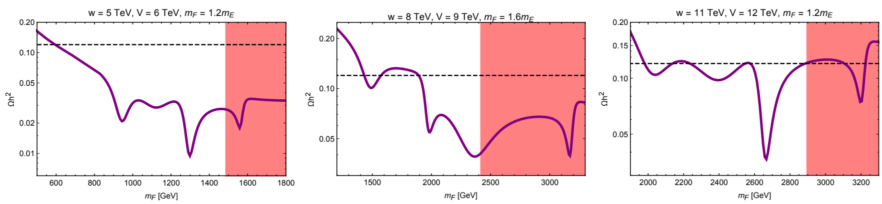

Fig 2.4: The total relic density of two component fermion dark matter as a function of dark matter masses for the case $m_F > m_E$.

In figure 2.4 we depict the total relic density as a function of m_F for several choices w, V and m_E as related to m_F, which are viable from the above contours.

In figure 2.5, we make a comparison between partial relic densities of dark matter component with the choices of the new physics scales w, V and m_E via m_F.

In figure 2.6, we plot the SI cross-sections of dark matter components corresponding to the above choices of (w, V) parameters, respectively.

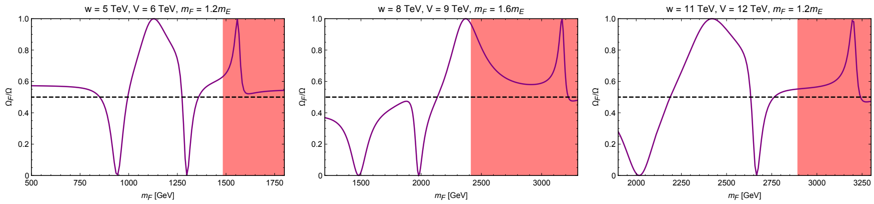

Fig 2.5: The contribution ratio of fermion dark matter components to the density as a function of dark matter masses for the case $m_F > m_E$.

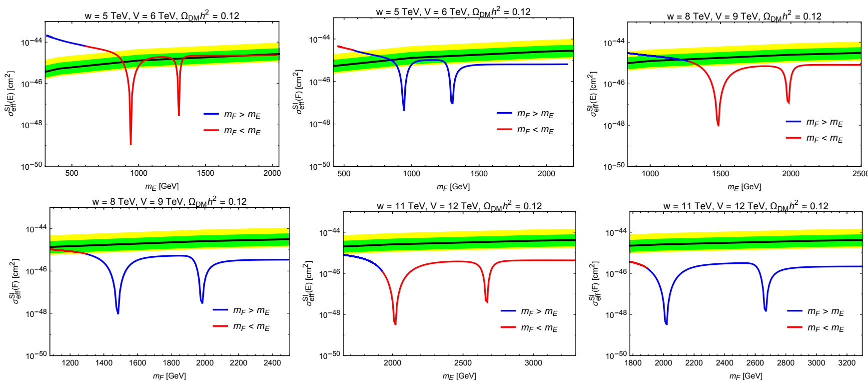

Fig 2.6: The spin-independent dark matter-nucleon scattering cross-section limits as a function of dark matter masses according to $(w, V) = (5, 6)$, (8,9), and (11,12) TeV , assuming the correct abundance: $\Omega_{\text{DM}}h^2 = 0.12$.

2.4.2. Scenario with two scalar dark matter

We consider the second case where two component dark matter contains the scalar particles $\mathcal{H}_2$, and $\mathcal{H}_3$. For this scenario, we have investigated and obtained results in the thesis: the dark matter pair annihilation into the standard model particles and conversion between dark matter component, the dark matter relic abundance of two dark matter, the direct detection for the dark matter components in our model through their spin-independent (SI) scattering on nuclei.

2.4.3. Scenario with a fermion and a scalar dark matter

In this case, we consider E và $\mathcal{H}_3$ to be the two component dark matter candidates. We have similarly investigated to the first and the second scenarios and clearly shown in the thesis.

2.5. Conclusion

We have shown that a gauge theory that includes a higher weak isospin symmetry $SU(P)_L$ must possess a complete gauge symmetry of the form $SU(3)_C \times SU(P)_L \times U(1)_X \times U(1)_N$, where the last two Abelian groups define the electric charge Q and baryon minus lepton charge $(B-L)$, respectively. The last charges are unified with the weak charge in the same manner as the electroweak theory. Additionally, the neutrino masses are appropriately induced by the gauge symmetry breaking, supplied in terms of a canonical seesaw mechanism.

The multiple matter parity P, $P = \bigotimes_{k=1}^{P-2} P_k$, where each P_k is a Z_2, is obtained as a residual gauge symmetry. Since this parity makes $(P-2)$ wrong particles stable, predicting the $(P-2)$ component dark matter candidates. The noncommutation of $(B-L)$ with $SU(P)_L$ yields that the dark matter candidates are nontrivially unified with normal matter in gauge multiplets. It means that multicomponent dark matter is required to complete the $SU(P)_L$ representations enlarged from the standard model. Therefore, the gauge interactions would govern the dark matter observables.

We study in detail the minimal multicomponent dark matter model corresponds to $P = 4$, the so-called $3-4-1-1$ model. The different versions of the $3-4-1-1$ model is give from choosing parameters β, γ varies. For instance, we have offer four versions corresponding to four choices β, γ varies. We found the version with $\beta = -\frac{1}{\sqrt{3}}, \gamma = -\frac{1}{\sqrt{6}}$, i.e $p = q = 0$, will predict many scenarios of two component dark matter candidates (there may exist three scenarios of two component dark matter). To study detail the two component dark matter characters, we searched all the interactions of fermions and scalars with gauge bosons, the physical particle spectrum of Higgs boson and their interactions with gauge bosons. The $3-4-1-1$ model with $q = p = 0$ obeys three possibilities of two component dark matter, including two fermions (E, F), two scalars $(\mathcal{H}_{2,3})$, and a fermion and a scalar $(E, \mathcal{H}_3)$, candidates, respectively. We have shown the viable parameter space for each scenario satisfying the relic density and direct detection. Typically, the dark matter masses are obtained around one to a few TeV. Additionally, there are four resonances in relic density set by the new neutral gauge $Z_{2,3}$ or the new neutral Higgs $H_{3,4}$ portals.

CHAPTER 3. THE EFFECT OF THE KINETIC MIXING TERM TO PHYSICS EFFECTS IN THE $3-4-1-1$ MODEL

3.1. The effect of the kinetic mixing parameter to the gauge field mass

3.1.1. Canonical basis of the kinetic mixing term

Lagrangian describes the kinetic of gauge fields, associating with two $U(1)$ groups of the $3-4-1-1$ model, including the kinetic mixing term among the $U(1)$ gauge fields. Let us we write down the kinetic terms of the two $U(1)$ gauge fields as:

$$\begin{aligned}\mathcal{L}_{\text{kinetic}} &\supset -\frac{1}{4}B_{\mu\nu}^2 - \frac{1}{4}C_{\mu\nu}^2 - \frac{\delta}{2}B_{\mu\nu}C^{\mu\nu} \\ &= -\frac{1}{4}(B_{\mu\nu}+\delta C_{\mu\nu})^2 - \frac{1}{4}(1-\delta^2)C_{\mu\nu}^2, \end{aligned} \tag{3.1}$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$ are the corresponding field strength tensors. The parameter δ is dimensionless, called the kinetic mixing term. Because of the kinetic mixing term (δ), the two gauge bosons B_μ and C_μ are generally not orthonormalized. We change to the canonical basis by a nonunitary transformation $(B_\mu, C_\mu) \to (B'_\mu, C'_\mu)$, where:

$$B' = B + \delta C, \qquad C' = \sqrt{1-\delta^2}C. \tag{3.2}$$

We substitute B, C in terms of $B', C,'$ into the covariant devivative. It becomes:

$$D_\mu \supset ig_X X B_\mu + ig_N N C_\mu = ig_X X B'_\mu + \frac{i}{\sqrt{1-\delta^2}}(g_N N - g_X X\delta)C'_\mu, \tag{3.3}$$

3.1.2. Gauge boson mass

The $3-4-1-1$ symmetry breaking leads to mixing among A_3, A_8, A_{15}, B', and C'. Their mass Lagrangian arises from $\sum_S (D_\mu\langle S\rangle)^\dagger(D^\mu\langle S\rangle)$, such that:

$$\mathcal{L}^{\text{neutral}}_{\text{mass}} = \frac{1}{2}\left(A_3\ A_8\ A_{15}\ B'\ C'\right) M^2 \left(A_3\ A_8\ A_{15}\ B'\ C'\right)^T , \qquad (3.4)$$

In summary, the original fields are related to mass eigenstates by U, $(A_3\, A_8\, A_{15}\, B\, C)^T = U(A\, Z_1\, Z_2\, Z_3\, Z_4)^T$. For the first case, $w, V \ll \Lambda$, we have $U = U_\delta U_1 U_2 U_3 U_\varphi \simeq U_\delta U_1 U_2 U_\varphi$. For the second case, $w \ll V, \Lambda$, we obtain $U = U_\delta U_1 U_2 U_3' U_\xi \simeq U_\delta U_1 U_2 U_\xi$. For the last case, $w, \Lambda \ll V$, the mixing matrix is $U = U_\delta U_1 U_2 U_3' U_\xi$. Here we define:

$$\begin{aligned} U_\delta &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{\delta}{\sqrt{1-\delta^2}} \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{1-\delta^2}} \end{pmatrix}, \quad U_\varphi = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & c_\varphi & s_\varphi & 0 \\ 0 & 0 & -s_\varphi & c_\varphi & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \\ U_\xi &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & c_\xi & s_\xi \\ 0 & 0 & 0 & -s_\xi & c_\xi \end{pmatrix}. \end{aligned} \qquad (3.5)$$

The fields A, Z_1 are identical as photon and neutral gauge boson Z of the standard model, respectively. Whereas Z_2, Z_3 and Z_4 are new, heavy gauge bosons. The mixings of standard model gauge bosons with the new gauge bosons are very small, while the mixing within the new gauge bosons may be large.

3.2. ρ parameter in the $3-4-1-1$ model

In section, we study the contribution of new physics and the effect of kinetic mixing parameter to the ρ-parameter in the $3-4-1-1$ model. The new physics that contributes to the ρ-parameter starts from the tree-level:

$$\begin{aligned} \Delta\rho &= \frac{m_W^2}{c_W^2 m_{Z_1}^2} - 1 \\ &= \frac{m_Z^2}{m_Z^2 - \epsilon_1 m^2_{ZZ'_2} - \epsilon_2 m^2_{ZZ'_3} - \epsilon_3 m^2_{ZC'}} - 1 \\ &\simeq \frac{\epsilon_1 m^2_{ZZ'_2} + \epsilon_2 m^2_{ZZ'_3} + \epsilon_3 m^2_{ZC'}}{m_Z^2} \\ &\equiv (\Delta\rho)^0 + (\Delta\rho)^\delta, \end{aligned} \qquad (3.6)$$

where:

$$(\Delta\rho)^0 \simeq \frac{1}{4[1+(\beta^2+\gamma^2)t_X^2]^2}\left\{\frac{[\beta_2 u^2 + (2\sqrt{3}\beta t_X^2 - \beta_2)v^2]^2}{(u^2+v^2)w^2}\right.$$

$$+\frac{\{(\beta+2\sqrt{2}\gamma)t_X^2(u^2+v^2)+\sqrt{3}[1+(\beta^2+\gamma^2)t_X^2](u^2-v^2)\}^2}{3(u^2+v^2)V^2}$$
$$\left.+\frac{(b\beta+c\gamma)^2t_X^4(u^2+v^2)}{4\Lambda^2}\right\}, \quad (3.7)$$
$$(\Delta\rho)^\delta \simeq \frac{\delta[\delta+2(b\beta+c\gamma)t_Xt_N]t_X^2(u^2+v^2)}{16[1+(\beta^2+\gamma^2)t_X^2]^2t_N^2\Lambda^2}. \quad (3.8)$$

In figure 3.1, we make a contour of $\Delta\rho$ as the function of (u,w) concerning the first case of VEV arrangement. Here, the panels arranging form left to right correspond to the four dark matter of the $3-4-1-1$ model such as: $(\beta=1/\sqrt{3},\gamma=1/\sqrt{6})$, $(\beta=1/\sqrt{3},\gamma=-\sqrt{2}/\sqrt{3})$, $(\beta=-1/\sqrt{3},\gamma=\sqrt{2}/\sqrt{3})$ và $(\beta=-1/\sqrt{3},\gamma=-1/\sqrt{6})$.

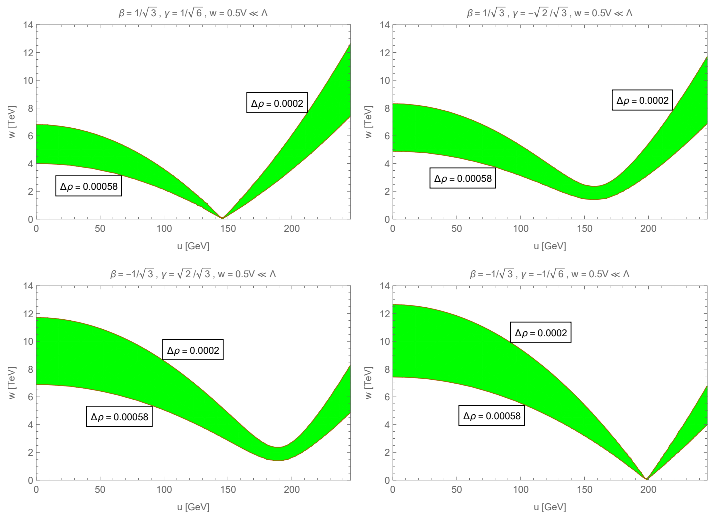

Fig 3.1: The (u,w) regime that is bounded by the ρ parameter for $w=0.5V\ll\Lambda$, where the panels from left to right and top to bottom correspond to the four dark matter models.

3.3. The effect of the kinetic mixing parameter to interaction of bosons Z_1 with fermions

We consider only the sensitivity of the new physics scales in terms of the kinetic parameter for case $(w,\Lambda\ll V)$. The results are given in figure 3.7. It indicates that the new physics regime changes when δ varies.

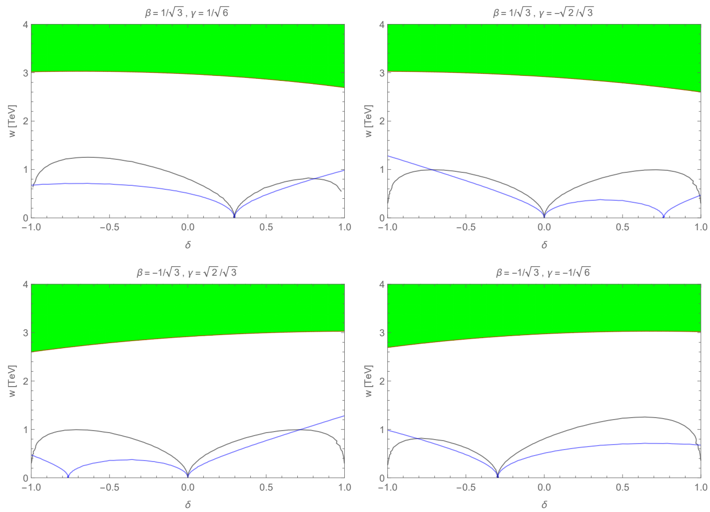

Fig 3.7: The bounds on new physics scales as the functions of δ for $|\epsilon_{1,2,3}| = 10^{-3}$, where the red, blue, and black lines correspond to ϵ_1, ϵ_2, ϵ_3 for the four kinds of dark matter models ($\beta = 1/\sqrt{3}, \gamma = 1/\sqrt{6}$), ($\beta = 1/\sqrt{3}, \gamma = -\sqrt{2}/\sqrt{3}$), ($\beta = -1/\sqrt{3}, \gamma = \sqrt{2}/\sqrt{3}$), and ($\beta = -1/\sqrt{3}, \gamma = -1/\sqrt{6}$), respectively.

3.4. Mesons oscillation in the $3-4-1-1$ model

The mass matrices of new fermions E_a, F_a, J_a, and K_a, which all have masses at w, V, and are given by:

$$[m_E]_{ab} = -h^E_{ab}\frac{w}{\sqrt{2}}, \qquad [m_F]_{ab} = -h^F_{ab}\frac{V}{\sqrt{2}}, \tag{3.9}$$

$$[m_J]_{33} = -h^J_{33}\frac{w}{\sqrt{2}}, \qquad [m_J]_{\alpha\beta} = -h^J_{\alpha\beta}\frac{w}{\sqrt{2}}, \tag{3.10}$$

$$[m_K]_{33} = -h^K_{33}\frac{V}{\sqrt{2}}, \qquad [m_K]_{\alpha\beta} = -h^K_{\alpha\beta}\frac{V}{\sqrt{2}}, \tag{3.11}$$

The mass matrices of charged-leptons and quarks e_a, u_a, and d_a are obtained:

$$\begin{aligned}
&[m_e]_{ab} = -h^e_{ab}\frac{v}{\sqrt{2}}, \\
&[m_u]_{3a} = -h^u_{3a}\frac{u}{\sqrt{2}}, \qquad [m_u]_{\alpha a} = h^u_{\alpha a}\frac{v}{\sqrt{2}}, \\
&[m_d]_{3a} = -h^d_{3a}\frac{v}{\sqrt{2}}, \qquad [m_d]_{\alpha a} = -h^d_{\alpha a}\frac{u}{\sqrt{2}},
\end{aligned} \tag{3.12}$$

which provide appropriate masses at u, v scale. For the neutrinos, $\nu_{aL,R}$, the Dirac and Majorana masses are $[m_\nu]_{ab} = -h^\nu_{ab}\frac{u}{\sqrt{2}}$ and $[m^R_\nu]_{ab} = -\sqrt{2}h'^\nu_{ab}\Lambda$, respectively. Since $u \ll \Lambda$, the observed neutrino ($\sim \nu_{aL}$) achieve masses via the type I seesaw mechanism,

$$m^L_\nu \simeq -m_\nu (m^R_\nu)^{-1}(m_\nu)^T \sim u^2/\Lambda. \tag{3.13}$$

which is small, as expected. The sterile neutrino ($\sim \nu_{aR}$) obtain large masses, such as (m^R_ν).

The meson mixing parameter is described via the effective interaction. In the $3-4-1-1$ model, the interaction of $Z_{1,2,3,4}$ vill affect the mixing among mesons, where the contribution Z_1 is small and omitted.

The strongest bound of the new physics comes from experiments of $B^0_s - \bar{B}^0_s$ oscillation.

$$[(V^*_{dL})_{32}(V_{dL})_{33}]^2\left(\frac{g_2^2}{m^2_{Z_2}} + \frac{g_3^2}{m^2_{Z_3}} + \frac{g_4^2}{m^2_{Z_4}}\right) < \frac{1}{(100\ \text{TeV})^2}. \tag{3.14}$$

In figure 3.8, we see that the new physics regime changes when δ varies.

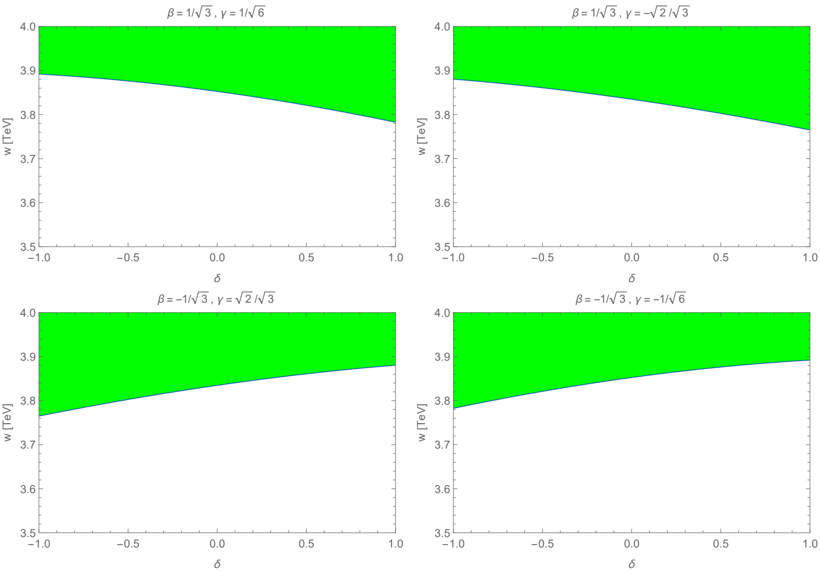

Fig 3.8: The bounds on the new physics scales as functions of δ from the FCNCs for $w = 0.5\Lambda \ll V$, where the panels from left to right and top to bottom are for the four dark matter models, respectively.

3.5. The physics at the colliders

Since the new neutral gauge bosons couple to leptons and quarks, they contribute to the Drell-Yan and dijet processes at colliders.

The LEPII searches for $e^+e^- \to \mu^+\mu^-$ happen similarly to the case of the $3-3-1-1$ model, where all the new gauge bosons $Z_{2,3,4}$ mediate the process. Assuming that all the new physics scales are the same order, they are bounded in the TeV scale.

The LHC searches for dijet and dilepton final states can be studied. Using the above condition, the new physics scales are also in TeV.

3.6. Conclution

We have studied the effect of the kinetic mixing parameter to some new physical effects in the $3-4-1-1$ model. We have proved that the $3-4-1-1$ model provides two component dark matter candidates naturally, supplying small neutrino masses via the seesaw mechanism induced

We have found that the kinetic mixing effects are evaluated, yielding the new physics scales at TeV scale, in agreement with the collision. To depend on the scheme of the gauge symmetry breaking, the effect of the kinetic mixing term to the new physic effect will be varies. In the $3-4-1-1$ model, there may exist manner to the symmetry breaking which may be cancel out only in the new gauge sector. We would like to emphasize, similar to the $3-3-1-1$, the $3-4-1-1$ model can address the question of cosmic inflation as well as asymmetric dark and normal matter.

GENERAL CONCLUSION

We studied the issues as follows:

- We construct particle structure of the $3-4-1-1$ model, ensuring the anomaly cancellation of gauge groups and obtaining residual gauge symmetry, $P = Z_2 \times Z_2$, is remnant of the gauge symmetry after spontaneous symmetry breaking. Therefore, we show that there may exist the two component dark matter in the model.

- We suggest the four versions corresponding to four the different selector of (p, q) parameter (i.e β, γ). And we find that the version with $p = q = 0$ has a rich two component dark matter candidates. In this case, the model obeys three possibilities of two component dark matter, including two fermions, two scalars, and a fermion and a scalar candidates.

- Studying the dark matter characters corresponding to version with $p = q = 0$. We based upon the imposing conditions for dark matter as relic density, the SI cross-section for direct searches are limit by experimental, we find that if the viable dark matter masses are around one to few TeV, which may coincide the imposing conditions as mentioned.

- Due to the appearance of the kinetic mixing term in the model, we will study the effect of the kinetic mixing parameter to some physic effects. Additionally, the kinetic mixing term will directly affect to mixing angle of the neutral gauge bosons. Hence, the mass spectrum of the gauge bosons in will change which there is directly influences to parameter ρ. Beside, the kinetic mixing parameter also affect to the coupling constant of neutral gauge bosons with fermions and anti fermions in the model. On the other hand, anomaly cancellation demands that the number of fermion quadru-plets equals that of fermion anti-quadru-plets, since a representation and it conjugate have opposite anomaly contributions which leads to the existence of FCNCs associated with neutral gauge bosons that these interactions are dominated by oscillation experiments of mesons. Therefore, we have studied the influence of the kinetic mixing parameter on the meson mixing parameters. We consider that, to depend on the hierarchy of VEVs take in the spontaneous symmetry breaking, the parameter do not effect to the physic effect as mentioned. It means that, the kinetic mixing effect are canceled out by the spontaneous symmetry breaking.

New finding of the thesis:

- We have shown that the $3-4-1-1$ model solves problems beyond the standard model, such as neutrino mass and dark matter, which attracts much attention by the scientist. We have proved that $3-4-1-1$ model provides neutrino mass naturally via the seesaw mechanism by the gauge symmetry after spontaneous symmetry breaking.

- We have shown that $3-4-1-1$ model is studied, which the kinetic mixing effects are considered. Because the new physical scale is chaned by the contribution of kinetic mixing, beside the interaction constant of the boson in SM is also changed by the mixing parameter.

LIST OF WORKS HAS BEEN PUBLISHED

1. Duong Van Loi, Phung Van Dong and Le Xuan Thuy, *Kinetic mixing effect in noncommutative* $B-L$ *gauge theory* , JHEP **09**, 2019, 054.

2. Cao Hoang Nam, Dương Van Loi, Le Xuan Thuy and Phung Van Dong, *Muticomponment dark matter in noncommutative* $B-L$ *gauge theoryt*, JHEP **12**, 2020, 029.

3. D. T. Huong, L. X. Thuy, N. T. Nhuan and H. T. Phuong, *Investigation of the FCNC processes in the* $3-4-1-1$ *model*, Communications in Physics, Vol. 31, No. 4 (2021), pp. 363-374.

In this thesis, I used first and second articles.