MINISTRY OF EDUCATION
AND TRAINING

VIETNAM ACADEMY OF
SCIENCE & TECHNOLOGY

GRADUATE UNIVERSITY OF SCIENCE AND TECHNOLOGY

- - - - - - - *** - - - - - - -

VU HOA BINH

SCALAR SECTOR PHENOMENOLOGY IN THE 3-3-1 MODEL WITH AN AXION LIKE PARTICLE

Major: Mathematical Physics and Theoretical Physics
Code: 9 44 01 03

SUMMARY OF DOCTORAL THESIS IN PHYSICS

SUPERVISOR 1: Assoc. Prof. Do Thi Huong
SUPERVISOR 2: Prof. Hoang Ngoc Long

HANOI - 2023

This thesis was completed at Graduate University of Science and Technology - Vietnam Academy of Science and Technology

Supervisor 1 : Assoc. Prof. Do Thi Huong,
Institute of Physics,
Vietnam Academy of Science and Technology
Supervisor 2 : Prof. Hoang Ngoc Long,
Institute of Physics,
Vietnam Academy of Science and Technology

Referee 1 :

Referee 2 :

Referee 3 :

The thesis was defensed at Graduated University of Science and Technology - Vietnam Academy of Science and Technology at ..., 202...

The thesis can be found at:
- Library of Graduate University of Science and Technology
- National Library of Vietnam

INTRODUCTION

Motivations of the thesis

Standard model (SM) is based on the $SU(3)_C$ group which describes the strong interaction and the $SU(2)_L \otimes U(1)_Y$ qroup which describes the electroweak interaction. SM success in describing three types of natural interactions: strong interaction, weak interaction and electromagnetic interaction. SM also predict successfully the existences of c, b, t quarks as well as Z, W bosons, especially Higgs boson with mass around 125GeV. The predictions were tested by experiments with high accuracy. SM explained almost of the experimental results about the micro-world. But there are still various experimental results which can not be explained by SM. These signals or data beyond of SM are considered signals of new physics (NP). The most prominent and typical are the experiments that determine the neutrino oscillations and experiments on cosmic background radiation analysis. On the other hand, SM itself does not have candidates for dark matter (DM) and dark energy (DE), but the analysis of the cosmic background radiation shows the existence of a large amount of DM and DE in the Universe. There also deviations between experimental measurements and SM's predictions, such as mass spliting of mesons, branching ratio of Higgs. The tolerances of these deviations is not large enough to be considered as NP but these can encourage the scientists who researche on beyond standard model (BSM). Besides of new experimental results, some problems such as the assymetry of matter - antimatter, number of fermion generations, the herachy of fermions' masses, the quantization of charge or the strong CP problem and etc are also needed to be solved.

These are the reasons why extending SM is naturally necessary. One can extend SM by extending symmetry groups or spectrum of particles or combining these two extending ways. External symmetry can be extended by adding extra dimesions of space or using supersymmetry model (SUSY). Internal symmetry can also be extended by extending the electroweak symmetry of SM. To solve either current problems of particles physics or uniform three types of interactions, one used single symmetry groups ($SU(5)$, $SU(10), E(6), E(8)$) whose Lie algebra includes $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ group. Now, BSMs should be improved to solve the problems which are out of SM. The 3-3-1 models are for example. At the beginning, the 3-3-1 models can explain why the number of fermion generation should be 3 as well as the problem of tiny mass of neutrino. The PQ symmetry naturally appears in 3-3-1 models then the SCPp can be solved [1, 2], etc... Moreover, the 3-3-1 model includes the axion-like particle (ALP) [3] which extremely tiny interact with ordinary matter. This particle can be a DM candidate.

The diversity of scalar sector of the 3-3-1 models is advantageous to study the existence of new Higgs with mass around 96GeV or 150GeV. There are some versions of 3-3-1 models such as the simple 3-3-1 model (S331), the economical 3-3-1 model (E331), etc ... The version ν331 which is imposed $Z_{11} \otimes Z_2$ discrete symmetry (A331) help us solve the SCPp with the prediction of the existence of DM axion [4,5]. In 2019, the XENON1T experiment showed that there might be an existence of light DM candidate with mass at keV scale. This may be DM axion. This is the reason why we were back to do research on A331 model and its scalar fields' properties. Being studied through 20 years, the A331 is still uncompleted because of the incorrection of diagonalizations of mass mixing matrices in neutral CP scalar sector. The physical states are incorrectly defined leads to some incorrect conclusions. Masses of Higgs boson in neutral CP even scalar sector are undefinded then one can not point a boson which can be uniformed with the Higgs boson of SM (we named this particle SM-like Higgs boson, SMLHB). Without the physical states and masses of particles of the A331 model, one can not study the phenomenology in this model. The pseudo scalar field in this model is almost as same as the QCD axion but it can interact with ordinary matter. This is completely different from QCD axion because QCD axion doesn't have Yukawa interaction with ordinary matter. So that we call this pseudoscalar axion-like particle (ALP) and the A331 is renamed ALP331. Our research was published in [3]. The reason why the $Z_{11} \otimes Z_2$ discrete symmetry was imposed was explained. We also corrected the rule of Z_2 symmetry to make it consistent. Then, we defined the physical states and the corresponding massess of fields (scalar fields and gauge fields) in the model. These helps us define the couplings of Higgs with gauge fields and matter field (quarks and leptons).

We have uniformed some particles and interactions of ALP331 with particles and interactions of SM. We also studied the contributions of NP in some processes such as meson oscillations, rare decays of t quark or decays of SMLHB and the new light Higgs (h_5) to find the reasonable constraints of some parameters in ALP331. By these above reason, we chose the project *"Scalar sector phenomenology in the 3-3-1 model with axion-like particle"*.

Research purposes

- Study on the scalar sector of ALP331 model to define the physical states and masses of the particles of the model. Then, uniform some particles and interactions with those in SM.
- Predict the existences of new particles such as ALP, a new light pseudoscalar, a new light Higgs with mass at EW scale, a very heavy Higgs with mass around 10^{11} GeV which can be an inflaton in Early Universe.
- Study on the contributions of NP in known processes like meson oscillations or rare t quark decays and decays of SMLHB.

Object and scope of the research

- The mass mixing matrices of scalar sector.
- Yukawa interactions with some mixing parameters in ALP331 model.
- Meson oscillations, decays of SMLHB h, a new light Higgs h_5 and rare decays of t quark by flavour changing neutral currents (FCNCs) in ALP331 model.

Research content

- Study the ALP331 to define exactly the physical states and corresponding masses of particles in the model.

- Analysize the contributions of NP in meson oscillations, rare t quark decays by FCNC and decays of SMLHB. Then find out the reasonable constraints of some parameters of the model.

Research methods

- Using quantum field theory, interactive field theory, Feynman rules, Feynman diagrams and Lie group theory, descrete symmetry groups,...
- Using Mathematica software.

Structure of the thesis

Excluding the introduction, conclusions and referrences, the main content of the thesis get 3 chapters.

Chapter 1. Overview: We present a brief review of SM and 3-3-1 models with its different versions. We also mention to an overview of QCD axion and axion-like particle (ALP). Besides, we give a brief review of meson oscillations, Higgs decays, rare *top* quark decays in SM.

Chapter 2. The 3-3-1 model with axion-like particle (ALP331): From the spectrum of particles and reasonable Yukawa interactions of the model, we explain the imposing of discrete symmetry $Z_{11} \otimes Z_2$ to ν331 model. This helps the PQ symmetry naturally appear. Then, we study for the details of gauge sector, Higgs sector with Higgs potential and Yukawa interactions to uniform some particles of ALP331 with particles of SM. By defining the couplings of fermions and Higgs bosons, we point out the constraints of mass mixing angles to make sure that the flavours can be preserved by interactions betweeen SMLHB and quarks at tree level. The numerical analysis of scalar sector helps us find out the constraints of some couplings and explain the very tiny value of λ_ϕ - the couplings of four scalar interaction.

Chapter 3. Higgs sector phenomenology in the ALP331: We study on decays of SMLHB (h) such as $h \to \bar{l}l$, $h \to \bar{b}b$, boson (h_5) and rare *top* quark decays $t \to hq$, $t \to q\gamma$ with ($q = u, c$) to find the constraints of some parameters about mass mixing angles or masses of the scalar fields in the ALP331. By studing meson oscillations, we define the lower limit of mass of some new light scalar fields.

Chapter 1. Overview

1.1. SM and scalar sector phenomenology in SM

1.1.1. The idea of SM

In the 20^{th} century, SM is considered the most successful theory of Particle Physics by using the $SU(3)_C$ group to describe the strong interaction in quantum chromodynamics (QCD) and $SU(2)_L \otimes U(1)_Y$ group (the Glashow - Weinberg - Salam model (GWS)) to describe the electroweak interaction. Before GWS model, one knew about weak interaction with two charge currents: $J_\mu = J_\mu^{had} + j_\mu^{lep}$. There was only one current in electromagnetic interaction: $J_\mu^{em} = \bar{\psi}_{(x)}^{(l)} \gamma_\mu \psi_{(x)}^{(l)} + \bar{\psi}_{(x)}^{(q)} \gamma_\mu \psi_{(x)}^{(q)}$, with l, q are the denotes of leptons and quarks. To describe these three currents, one need a symmetry group with at least three generators. The simplest symmetry group which can be used is $SU(2)$. But weak charge and charge corresponding to currents in J_μ are defined as below:

$$\begin{aligned} T_+(t) &= \frac{1}{2}\int d\vec{x} \left(j_0^{lep}(x) + j_0^{had}(x) \right) , \qquad T_-(t) = (T_+(t))^\dagger , && (1.1) \\ Q(t) &= \int d\vec{x} J_0^{em}(x) = -\int d\vec{x} \left\{ \left(\psi^{(l)}(x) \right)^\dagger \psi^{(l)}(x) + \left(\psi^{(q)}(x) \right)^\dagger \psi^{(q)}(x) \right\} && (1.2) \end{aligned}$$

so that $T_+(t), T_-(t)$ and $Q(t)$ are not in form of closed algebra because of $[T_+, T_-] = 2T_3$, with T_3 is:

$$T_3 = \int d\vec{x} \left\{ \left(\psi_L^{(l)}(x) \right)^\dagger \psi_L^{(l)}(x) + \left(\psi_L^{(\nu_l)}(x) \right)^\dagger \psi_L^{(\nu_l)}(x) + \left(\psi_L^{(q)}(x) \right)^\dagger \psi_L^{(q)}(x) \right\} . \quad (1.3)$$

To close the algebra of currents, one added an $U(1)$ symmetry. This is the reason why one use $SU(2)_L \otimes U(1)_Y$ symmetry group in electroweak theory. The charge operator is: $Q = T_3 + \frac{Y}{2}$. with T_3 is a diagonal generator of $SU(2)_L$ and Y is hyper-charge.

There is not FCNC of quarks and leptons in SM. All the currents of interactions are conserved at tree level. In the EW sector, fermions and bosons $(W^\pm, Z, A, h)$ are unsatisfied the DM properties. In the QCD sector, the

$SU(3)_C$ group has got eight generators so that one have to introduce eight gauge fields - gluons into the model. These gluons carry strong interactions. Without SSB of $SU(3)_C$, gluons are massless. QCD theory is tested by experiments with very high accuracy. When studing the current problems of particle physics, one usually keep $SU(3)_C$ group unchanged and extend the gauge symmetry group of EW sector.

SM successed in predicting the existences of s, b, t quarks and $W^{\pm}, Z$ bosons, espcially Higgs boson with mass around 125GeV. Not only problems of neutrinos' masses, DM and DE but also problems of number of fermions generation, assymmetry of matter-antimatter and some abnomarl results from the experiments are unsolved by SM.

1.1.2. Meson oscillations in SM

Neutral mesons are the combinations of same type quarks and anti-quark $(K^0, \bar{K}^0)$, $(B_s, \bar{B}_s)$, $(B_d, \bar{B}_d)$. The interaction of charge currunts can be predicted by the oscillations of $(K^0 - \bar{K}^0)$, $(B_s - \bar{B}_s)$,$(B_d - \bar{B}_d)$ as in the diagrams (1.1),(1.2) và (1.3).

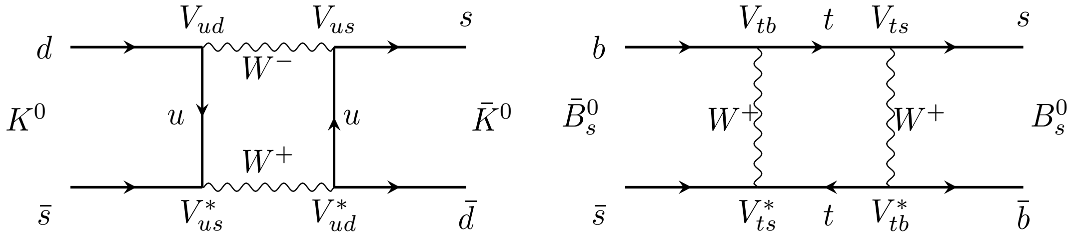

Figure 1.1: Box diagram of $K^0 - \bar{K}^0$ oscillations by interactions of charge currents in SM.

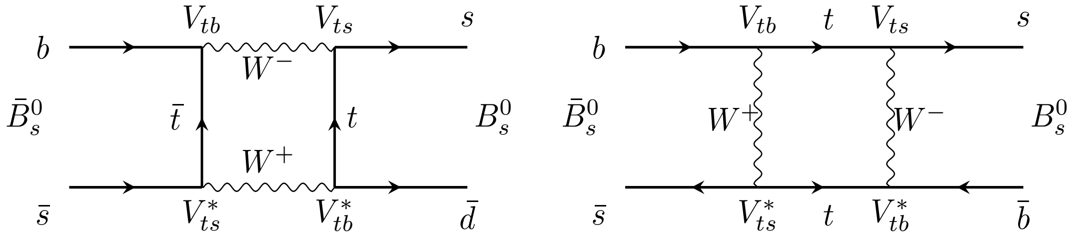

Figure 1.2: Box diagram of $B_s - \bar{B}_s$ oscillations by interactions of charge currents in SM.

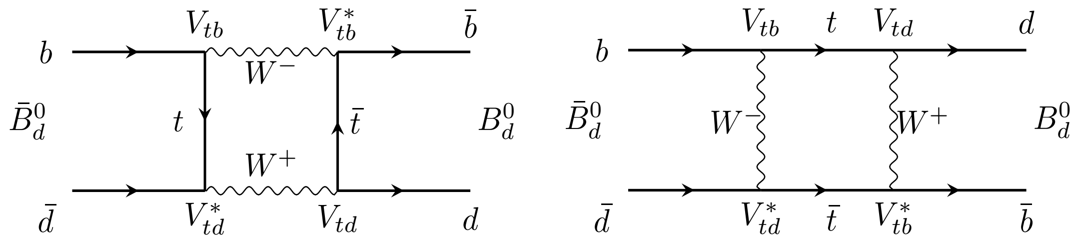

Figure 1.3: Box diagram of $B_d - \bar{B}_d$ oscillations by interactions of charge currents in SM.

In comparision with experimental data, the predictions of SM got some small deviations. These can be tolerances of definding the elements of V_{CKM} matric or tolerances of definding bound couplings f_M in QCD theory at low energy or caused of missing some next to leading order contributions of SM.

1.1.3. Decays of Higgs into two fermions in SM

The decay width of Higgs $h \to \bar{f}f$ with $f = u, d, c, s, b, \tau, \mu, e$ is:

$$\Gamma(h \to \bar{f}f) = \int d\Gamma = \frac{g^2_{(h,f,f)}}{8\pi} m_h \left(1 - \frac{4m_f^2}{m_h^2}\right)^{\frac{3}{2}} . \qquad (1.4)$$

1.1.4. Rare top quark decays in SM

In SM, t quark is the heaviest particle ($\sim$ 172 GeV) which means $m_t > m_h > m_Z > m_W$. Since, decays of t quark can be $t \to qh$, $t \to qZ$, $t \to q\gamma$ with $q = u, c$ and $t \to q'W$ with $q' = d, s, b$. These decays can appear at tree level by charge currents. But by neutral currents, the interactions between Higgs and fermions should be conserved at one loop level. Then, $t \to qh$, $t \to qZ$, $t \to q\gamma$ happen by radiative mechanism. This is the reason why these decays called rare decays. Some rare decays are $t \to qh$ and $t \to q\gamma$ with $q = u, c$ with very tiny branching ratios. These helps us to impose some constraints on new interactions of BSMs.

1.2. CP violation in strong interaction

Studying on vacuum's structure of QCD, one recognized the appearence of an invarriant term which violate the CP symmetry. This term has form as below:

$$G \cdot \tilde{G} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} G_a^{\mu\nu} G^{a\alpha\beta}\,, \tag{1.5}$$

where, $\tilde{G}$ is dual field strength tensor and defined as:

$$\tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta,a}\,. \tag{1.6}$$

This CP violated term is really a problem of QCD because until now, experiments couldn't dectect any signals fo CP violation in QCD. Theoretically, this CP violated term gives contributions to EDM of neutron. The EDM of neutron is undeteted that gives the upper limit of d_N with confident level 90% is $d_N \leqslant 2.9 \times 10^{-26} e\,\text{cm}$. This constraint required the fine turning parameter θ which is associated with $G \cdot \tilde{G}$, be very tiny. To make theory consistent with experiment data, $\theta \ll \mathcal{O}(10^{-9})$. How to explain the tiny of θ is called Strong CP problem - SCPp. SCPp can be solved by $U(1)_{PQ}$ symmetry with the appearence of a pseudoscalar axion QCD after SSB $U(1)_{PQ}$ global symmetry at $f_a \sim 10^{11}$ GeV scale. Axion-like particle (ALP) appears after SSB $U(1)$ global symmetry. ALP's potential and effective Lagrangian are as same as those of axion QCD but mass of ALP doesn't get from non-perturbative QCD effects. And ALP doean't interact with gluons as axion QCD does. Hence, ALP can not solve the SCPp.

1.3. The 3-3-1 models

The $3-3-1$ models are based on $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ symmetry group. The charge operator has form: $Q = T_3 + \beta T_8 + X$. The 3-3-1 models are classified by values of β.

1.3.1. The 3-3-1 model with $\beta = \pm\sqrt{3}$

The minimal 3-3-1 model (M331)

Quark sector: the first two generations are triplets $SU(3)_L$, the third generation is anti-triplet $SU(3)_L$. Scalar sector: requires three triplets and a sextet. Lepton sector: charge lepton is introduced to the bottom of anti-triplet lepton. The Higgs potential is complicated with various parameters. The model gets Landau pole.

The simple 3-3-1 model (S331)

Quark sector: The first generation is triplets $SU(3)_L$, the next two generations are anti-triplets $SU(3)_L$. Scalar sector: is the same as E331 model's. Lepton sector: is the same as M331 model's. The FCNC could be tiny because of independence of Landau pole. If one introduces some sterile scalar multiplets, there will DM candidates in the model.

The reduced minimal 3-3-1 model (RM331)

Quark sector: is as same as ν331 model's. Scalar sector includes two triplets χ, ρ. Lepton sector: is as same as M331 model's. The model gets Landau pole so that the contributions to FCNC is large and this is not consistent with data from experiments.

1.3.2. The 3-3-1 model with $\beta = \pm\frac{1}{\sqrt{3}}$

The 3-3-1 model with right-handed neutrino (ν331)

Quark sector: the first two generations are anti-triplets $SU(3)_L$, the third generation is triplet $SU(3)_L$. Scalar sector includes three triplets χ, ρ, η with two triplets (χ, η) have the same quantum numbers. Lepton sector: right-handed neutrino is introduced to the bottom of triplet lepton. There are an unnatural hierarchy and unstable DM particle with mass around MeV.

The economical 3-3-1 model (E331)

The spectrum of quark and lepton is as same as M331 model but the scalar sector is the one reduced from ν331's. There are two triplets have same quantum number but number of lepton. If number of lepton is violated, these two scalars are extremely equivalent, hence one can remove a scalar. The two scalar in E331 model are χ, η. E331 has least number of multiplets, the amount of independent parameters is less than the other 3-3-1 models.

The 3-3-1 model with axion (A331)

The spectrum of particles of A331 model includes all the particles of ν331 model and a singlet complex scalar $\phi \sim (1, 1, 0)$ with VEV $v_\phi \sim 10^{11}$ GeV.

The gauge symmetry of the model is $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_{11} \otimes Z_2$. The physical state of axion [4] is:

$$a = \frac{1}{\sqrt{1+\frac{v^2_{\chi'}}{v^2_{\phi}}}}(I_\phi + \frac{v_{\chi'}}{v_\phi} I_{\chi'})\,. \tag{1.7}$$

From Eq. (1.7), axion a is a combination of two components $I_\phi, I_{\chi'}$ hence, it doesn't have Yukawa interactions with ordinary.

1.4. Conclusion of chapter 1

The idea of building SM and SM's properties are presented basically. FCNCs do not exist at tree-level but appeare by one loop radiative contribution in meson ocsillation. SM Higgs decays at tree and one loop level are also presented to analize the branching ratios of SMLHB and some rare decays such as $t]tohq, t \to q\gamma$ with $q = u, c$.

Chapter 2. The scalar sector of $3-3-1$ model with axion-like particle

2.1. Particle content

Spectrum of particles in the ALP331 model includes all the particles of the $\nu331$ model and a singlet scalar field $\phi \sim (1,1,0)$. To give masses to gauge bosons and fermions, these scalar fields must have non-zero VEVs.

2.2. The reason of imposing the discrete symmetry $Z_{11} \otimes Z_2$ on ν331 model and the natural appearence of PQ symmetry in the A331 model

With three scalar triplets χ, η, ρ, the Yukawa interactions (includes 12 multiplets) raise masses to fermions but incompleted. We introduce one more scalar singlet ϕ to provide masses to all the be left particles of the model. The full Yukawa interactions (includes 13 multiplets) are:

$$\begin{aligned} -\mathcal{L}_Y &= y_1 \bar{Q}_{3L} \chi U_R + y2 \bar{Q}_{aL} \chi^* D_{aR} + y_3 \bar{Q}_{3L} \eta u_{aR} + y_4 \bar{Q}_{aL} \eta^* d_{aR} \\ &+ y_5 \bar{Q}_{3L} \rho d_{aR} + y_6 \bar{Q}_{aL} \rho^* u_{aR} + h_{ab} \bar{\psi}_{aL} \rho l_{bR} + h^{'}_{ab} \epsilon^{ijk} (\bar{\psi}_{aL})_i (\psi_{bL})^c_j (\rho^*)_k \\ &+ (y_N)_{ab} \bar{N}^C_{aR} \phi N_{bR} + h.c\,, \end{aligned} \tag{2.1}$$

and the Higgs potential which raise masses to gauge bosons of the model is:

$$V_{Higgs} = V^{hermite}_{Higgs} + V^{non-hermite}_{Higgs}\,, \tag{2.2}$$

The condition of Z_N charge is given by inital Yukawa interactions. Hence, the Z_N symmetry should be with $N > 12$ inspite of the fact that there are 13 multiplets in the model. Since 12 is not prime, all phases of the multiplets will be converted to smaller discrete symmetries (Z_2, Z_3, Z_4, Z_6). This makes the higher order operators of ϕ are difficult to be vanished. So, the largest discrete symmetry which can be used is Z_{11} to forbid most terms in the $V_{Higgs}^{non-hermite}$. Only three terms $\chi^\dagger\eta\phi^*\phi^*, \eta\rho\chi\phi, \eta\eta\rho\phi^*$ are invarriant under Z_{11} transformation. To avoid the unwanted terms like $(\chi^\dagger\eta+\eta^\dagger\chi)^2$, Z_2 symmetry is imposed to vanish these terms. Under the rule of Z_2, the $SU(3)_L$ triplets η and χ get the opposite Z_2 charge. These below fields carry odd Z_2 charge:

$$(\eta\,,\rho\,,u_R\,,d_{nR},e_{nR},N_R) \quad \to \quad -(\eta\,,\rho\,,u_R\,,d_{nR},e_{nR},N_R)\,. \tag{2.3}$$

Imposing Z_2 symmetry on three Z_{11} invarriant terms, there is only non-hermitian term of $\eta\rho\chi\phi$ left with hermitian term in the Higgs potential. To provide masses to Dirac neutrino ν_L and Majorana neutrino N_R, the spectrum of particles of the ALP331 should have charge assignments as in the Table 2.1.

	Q_{nL}	Q_{3L}	u_{aR}	d_{aR}	U_{3R}	D_{nR}	ψ_{aL}	e_{aR}	N_{aR}	η	χ	ρ	ϕ
$SU(3)_C$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(3)_L$	$\overline{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
$U(1)_X$	0	$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	-1	0	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	0
Z_{11}	ω_4^{-1}	ω_0	ω_5	ω_2	ω_3	ω_4	ω_1	ω_3	ω_5^{-1}	ω_5^{-1}	ω_3^{-1}	ω_2^{-1}	ω_1^{-1}
Z_2	1	1	-1	-1	1	1	1	-1	-1	-1	1	-1	1

Table 2.1: $SU(3)_C \times SU(3)_L \times U(1)_X \times Z_{11} \times Z_2$ charge assignments of the particle content of the model with $a = 1, 2, 3$ and $\alpha = 1, 2$.

The PQ symmetry appears automatically if one assign the PQ charge to the particles as below: the left-handed quarks get the opposite PQ charge assignments with the right-handed quarks. Suppose that chiral fermions get opposite PQ charges and $X_d = X_D = 1$, we summarize the PQ charges of fermions in Table 2.2.

	u_{aL}	d_{aL}	U_L	$D_{\alpha L}$	ψ_{aL}	e_{aR}	ν_{aL}	ν_{aR}	N_{aR}
X_{PQ}	-1	1	1	1	1	1	1	-1	1

Table 2.2: PQ charges of fermions in the ALP331 model.

2.3. Gauge bosons

Nine gauge bosons arise from $SU(3)_L \otimes U(1)_X$ symmetry and get masses from the Lagrangian which includes kinertic terms. Their physical states are:

$$W_\mu^\pm \;=\; \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right), \qquad Y_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^6 \pm iW_\mu^7\right),$$

$$X^0_\mu = \frac{1}{\sqrt{2}}\left(W^4_\mu - iW^5_\mu\right), \quad X^{0*}_\mu = \frac{1}{\sqrt{2}}\left(W^4_\mu + iW^5_\mu\right). \tag{2.4}$$

After SSB, the spectrum of masses of gauge bosons are:

$$m^2_W = \frac{g^2}{4}(v^2_\eta + v^2_\rho), \quad m^2_{X^0} = \frac{g^2}{4}(v^2_\chi + v^2_\eta), \quad m^2_Y = \frac{g^2}{4}(v^2_\chi + v^2_\rho). \tag{2.5}$$

The W boson of the model is as same as the W boson in SM. The X^0 and Y boson are pairs of new gauge vector fields. These are heavy particles and get the mass spliting as: $|m^2_Y - m^2_{X^0}| < m^2_W$. The physical states defined by the mixing of W^3_μ, W^8_μ, B_μ are:

$$\begin{aligned} A_\mu &= s_W W_{3\mu} + c_W\left(-\frac{t_W}{\sqrt{3}}W_{8\mu} + \sqrt{1-\frac{t^2_W}{3}}B_\mu\right), \\ Z_\mu &= c_W W_{3\mu} - s_W\left(-\frac{t_W}{\sqrt{3}}W_{8\mu} + \sqrt{1-\frac{t^2_W}{3}}B_\mu\right), \\ Z'_\mu &= \sqrt{1-\frac{t^2_W}{3}}W_{8\mu} + \frac{t_W}{\sqrt{3}}B_\mu. \end{aligned} \tag{2.6}$$

2.4. Higgs potential

The scalar potential of the model has the form:

$$\begin{aligned} V &= \mu^2_\phi \phi^*\phi + \mu^2_\chi \chi^\dagger\chi + \mu^2_\rho \rho^\dagger\rho + \mu^2_\eta \eta^\dagger\eta + \lambda_1(\chi^\dagger\chi)^2 + \lambda_2(\eta^\dagger\eta)^2 \\ &+\lambda_3(\rho^\dagger\rho)^2 + \lambda_4(\chi^\dagger\chi)(\eta^\dagger\eta) + \lambda_5(\chi^\dagger\chi)(\rho^\dagger\rho) + \lambda_6(\eta^\dagger\eta)(\rho^\dagger\rho) \\ &+\lambda_7(\chi^\dagger\eta)(\eta^\dagger\chi) + \lambda_8(\chi^\dagger\rho)(\rho^\dagger\chi) + \lambda_9(\eta^\dagger\rho)(\rho^\dagger\eta) \\ &+\lambda_{10}(\phi^*\phi)^2 + \lambda_{11}(\phi^*\phi)(\chi^\dagger\chi) + \lambda_{12}(\phi^*\phi)(\rho^\dagger\rho) \\ &+\lambda_{13}(\phi^*\phi)(\eta^\dagger\eta) + \left(\lambda_\phi \epsilon^{ijk}\eta_i\rho_j\chi_k\phi + H.c.\right) \end{aligned} \tag{2.7}$$

The vacuum expectation values (VEV) v_ϕ triggers SSB Z_{11} which causes of the appearence of ALP at high energy scale $\sim 10^{10} - 10^{11}$ GeV. The minimum constraints of Higgs potentials at tree level as below:

$$\begin{aligned} \mu^2_\rho + \lambda_3 v^2_\rho + \frac{\lambda_5}{2}v^2_\chi + \frac{\lambda_6}{2}v^2_\eta + \frac{\lambda_{12}}{2}v^2_\phi + \frac{A}{2v^2_\rho} &= 0, \\ \mu^2_\eta + \lambda_2 v^2_\eta + \frac{\lambda_4}{2}v^2_\chi + \frac{\lambda_6}{2}v^2_\rho + \frac{\lambda_{13}}{2}v^2_\phi + \frac{A}{2v^2_\eta} &= 0, \\ \mu^2_\chi + \lambda_1 v^2_\chi + \frac{\lambda_4}{2}v^2_\eta + \frac{\lambda_5}{2}v^2_\rho + \frac{\lambda_{11}}{2}v^2_\phi + \frac{A}{2v^2_\chi} &= 0, \end{aligned}$$

$$\mu_\phi^2 + \lambda_{10} v_\phi^2 + \frac{\lambda_{11}}{2} v_\chi^2 + \frac{\lambda_{12}}{2} v_\rho^2 + \frac{\lambda_{13}}{2} v_\eta^2 + \frac{A}{2v_\phi^2} \quad = \quad 0\,, \tag{2.8}$$

where, $A \equiv \lambda_\phi v_\phi v_\chi v_\eta v_\rho$.

2.5. Charged scalar sector

In the basis (η_2^-, ρ_1^-), the squared mass mixing matrix is:

$$M_{c_1}^2 = -\frac{(A - \lambda_9 v_\rho^2 v_\eta^2)}{2} \begin{pmatrix} \frac{1}{v_\eta^2} & \frac{1}{v_\eta v_\rho} \\ \frac{1}{v_\eta v_\rho} & \frac{1}{v_\rho^2} \end{pmatrix}. \tag{2.9}$$

With $\tan\alpha = \frac{v_\eta}{v_\rho}$, the physical states are:

$$\begin{pmatrix} G_1^\pm \\ H_1^\pm \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \rho_1^\pm \\ \eta^\pm \end{pmatrix}, \tag{2.10}$$

In the basis (χ_2^-, ρ_3^-), the squared mass mixing matrix is:

$$M_{c2}^2 = -\frac{(A - \lambda_8 v_\rho^2 v_\chi^2)}{2} \begin{pmatrix} \frac{1}{v_\chi^2} & \frac{1}{v_\chi v_\rho} \\ \frac{1}{v_\chi v_\rho} & \frac{1}{v_\rho^2} \end{pmatrix}. \tag{2.11}$$

With $\tan\theta_1 = \frac{v_\rho}{v_\chi}$, the physical states are:

$$\begin{pmatrix} G_2^\pm \\ H_2^\pm \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \chi_2^\pm \\ \rho_3^\pm \end{pmatrix}, \tag{2.12}$$

2.6. ALP in the ALP331 model

In the basis (I_χ^1, I_η^3), there are a massless scalar field G_1 and a massive scalar B_1 with mass is:

$$m_{B_1}^2 = -\frac{(A - \lambda_7 v_\eta^2 v_\chi^2)}{2} . \frac{(v_\eta^2 + v_\chi^2)}{v_\eta^2 v_\chi^2} . \tag{2.13}$$

The squared mass mixing matrix in the basis $(I_\phi, I_\chi^3, I_\rho, I_\eta^1)$ has form:

$$M_{odd}^2 = -\frac{A}{2} \begin{pmatrix} \frac{1}{v_\phi^2} & \frac{1}{v_\phi v_\chi} & \frac{1}{v_\phi v_\rho} & \frac{1}{v_\phi v_\eta} \\ & \frac{1}{v_\chi^2} & \frac{1}{v_\chi v_\rho} & \frac{1}{v_\chi v_\eta} \\ & & \frac{1}{v_\rho^2} & \frac{1}{v_\eta v_\rho} \\ & & & \frac{1}{v_\eta^2} \end{pmatrix}, \tag{2.14}$$

and is exactly diagonalized by Euler method. The physical fields are:

$$\begin{pmatrix} a \\ G_{Z'} \\ G_Z \\ A_5 \end{pmatrix} = \begin{pmatrix} c_{\theta_\phi} & -s_{\theta_3}s_{\theta_\phi} & -s_\alpha c_{\theta_3}s_{\theta_\phi} & -c_\alpha c_{\theta_3}s_{\theta_\phi} \\ 0 & c_{\theta_3} & -s_\alpha s_{\theta_3} & -c_\alpha s_{\theta_3} \\ 0 & 0 & c_\alpha & -s_\alpha \\ s_{\theta_\phi} & s_{\theta_3}c_{\theta_\phi} & s_\alpha c_{\theta_3}c_{\theta_\phi} & c_\alpha c_{\theta_3}c_{\theta_\phi} \end{pmatrix} \begin{pmatrix} I_\phi \\ I_\chi^3 \\ I_\rho \\ I_\eta^1 \end{pmatrix}, \quad (2.15)$$

in which, $\cos\xi = c_\xi$, $\sin\xi = s_\xi$ with $\xi = \alpha, \theta_3, \theta_\phi$ and the mixing angles are:

$$\tan\alpha = \frac{v_\eta}{v_\rho}, \qquad \tan\theta_3 = \frac{v_\eta}{v_\chi\sqrt{1+\frac{v_\eta^2}{v_\rho^2}}} \approx \frac{v_\eta}{v_\chi},$$
$$\tan\theta_\phi = \frac{v_\chi}{v_\phi\sqrt{1+v_\chi^2\left(\frac{1}{v_\rho^2}+\frac{1}{v_\eta^2}\right)}} \approx \frac{v_\chi}{v_\phi}. \quad (2.16)$$

The ALP is a massless particle which combines four components I_ϕ, I_χ^3, I_ρ cùng I_η^1:

$$a = I_\phi c_{\theta_\phi} - I_\chi^3 s_{\theta_\phi}s_{\theta_3} - I_\rho c_{\theta_3}s_\alpha s_{\theta_\phi} - I_\eta^1 c_\alpha c_{\theta_3}s_{\theta_\phi}. \quad (2.17)$$

This expansion is completely different from the axion a given in previous publications. The matrix (2.14) is diagonalized by the matrix in (2.15) with three mixing angles $\alpha, \theta_3, \theta_\phi$ defined by (2.16) and a parameter $\left(\frac{1}{v_\phi^2}+\frac{1}{v_\chi^2}+\frac{1}{v_\rho^2}+\frac{1}{v_\eta^2}\right)$ which is included in the equation of A_5's mass:

$$m_{A_5}^2 = -\frac{A}{2}\left(\frac{1}{v_\phi^2}+\frac{1}{v_\chi^2}+\frac{1}{v_\rho^2}+\frac{1}{v_\eta^2}\right) \approx -\frac{\lambda_\phi v_\phi v_\chi}{\sin 2\alpha}. \quad (2.18)$$

From the equation (2.18), the value of λ_ϕ should be negative.

2.7. *CP* even scalar sector

In the basis (R_χ^1, R_η^3), there are a massless scalar field G_4 and a massive scalar field H_1 with mass as:

$$m_{H_1}^2 = -\frac{(A-\lambda_7 v_\eta^2 v_\chi^2)}{2}.\frac{(v_\eta^2+v_\chi^2)}{v_\eta^2 v_\chi^2}. \quad (2.19)$$

In the basis $(R_\eta^1, R_\rho, R_\chi^3, R_\phi)$, the squared mass mixing matrix M_R^2 has form:

$$\begin{pmatrix} 2\lambda_2 v_\eta^2 - \frac{A}{2v_\eta^2} & \lambda_6 v_\eta v_\rho + \frac{\lambda_\phi v_\chi v_\phi}{2} & \lambda_4 v_\eta v_\chi + \frac{\lambda_\phi v_\rho v_\phi}{2} & \lambda_{13} v_\eta v_\phi + \frac{\lambda_\phi v_\rho v_\chi}{2} \\ \lambda_6 v_\eta v_\rho + \frac{\lambda_\phi v_\chi v_\phi}{2} & 2\lambda_3 v_\rho^2 - \frac{A}{2v_\rho^2} & \frac{\lambda_\phi v_\eta v_\phi}{2} + \lambda_5 v_\rho v_\chi & \frac{\lambda_\phi v_\eta v_\chi}{2} + \lambda_{12} v_\rho v_\phi \\ \lambda_4 v_\eta v_\chi + \frac{\lambda_\phi v_\rho v_\phi}{2} & \frac{\lambda_\phi v_\eta v_\phi}{2} + \lambda_5 v_\rho v_\chi & 2\lambda_1 v_\chi^2 - \frac{A}{2v_\chi^2} & \frac{\lambda_\phi v_\eta v_\rho}{2} + \lambda_{11} v_\chi v_\phi \\ \lambda_{13} v_\eta v_\phi + \frac{\lambda_\phi v_\rho v_\chi}{2} & \frac{\lambda_\phi v_\eta v_\chi}{2} + \lambda_{12} v_\rho v_\phi & \frac{\lambda_\phi v_\eta v_\rho}{2} + \lambda_{11} v_\chi v_\phi & 2\lambda_{10} v_\phi^2 - \frac{A}{2v_\phi^2} \end{pmatrix} \quad (2.20)$$

Comparing with the corresponding matrix in [4,5], we see that the first three elements in the forth column should have the extra terms $\frac{\lambda_{11} v_\phi v_{\chi'}}{2}$, $\frac{\lambda_{13} v_\phi v_\eta}{2}$ and $\frac{\lambda_{12} v_\phi v_\rho}{2}$. The matrix which is used to diagonalize the matrix M_R^2 is:

$$U_R = \begin{pmatrix} -c_{\alpha_2} & -s_{\alpha_2} c_{\alpha_3} & -s_{\alpha_2} s_{\alpha_3} c_{\alpha_\phi} & s_{\alpha_2} s_{\alpha_3} s_{\alpha_\phi} \\ s_{\alpha_2} & -c_{\alpha_2} c_{\alpha_3} & -c_{\alpha_2} s_{\alpha_3} c_{\alpha_\phi} & c_{\alpha_2} s_{\alpha_3} s_{\alpha_\phi} \\ 0 & s_{\alpha_3} & -c_{\alpha_3} c_{\alpha_\phi} & c_{\alpha_3} s_{\alpha_\phi} \\ 0 & 0 & s_{\alpha_\phi} & c_{\alpha_\phi} \end{pmatrix}, \tag{2.21}$$

where, the mixing angles of the CP even scalar sector are :

$$\tan 2\alpha_2 = \frac{4 c_{\alpha_3} v_\eta v_\rho (A + \lambda_6 v_\eta^2 v_\rho^2)}{A c_{\alpha_3}^2 v_\eta^2 - A v_\rho^2 + 4 v_\eta^2 v_\rho^2 (\lambda_2 v_\eta^2 - \lambda_3 c_{\alpha_3}^2 v_\rho^2)}, \tag{2.22}$$

$$\tan 2\alpha_3 = \frac{4 v_\chi \left(A + 2\lambda_5 v_\rho^2 v_\chi^2\right)}{c_{\alpha_\phi} \left(A - 4\lambda_1 v_\chi^4\right)^2}, \tag{2.23}$$

$$\tan 2\alpha_\phi = \frac{\lambda_{11} v_\chi}{\lambda_{10} v_\phi}. \tag{2.24}$$

We can uniform h with Higgs boson of SM. And h_5 is a new light Higgs with mass at EW scale, H_χ is a heavier Higgs with mass depends on v_χ. The new Higgs Φ with a very huge mass at v_ϕ scale and can be considered as an inflaton in the Early universe.

2.8. Numerical analysis of the scalar sector

1. Charged scalar sector: $\lambda_9 v_\rho^2 v_\eta^2 > A$ and $\lambda_8 v_\rho^2 v_\chi^2 > A$.

2. CP odd scalar sector: $\lambda_7 v_\eta^2 v_\chi^2 > A$. If $v_\eta = v_\rho$ in EW scale, then $\left(m_{A_5}^2\right)_{min} = -\lambda_\phi v_\phi v_\chi$. From the equation (2.18), one get $\lambda_\phi = -\frac{m_{A_5}^2 \sin 2\alpha}{v_\phi v_\chi}$. With $m_{A_5} \sim 10^3$ GeV, $v_\phi \sim 10^{10}$ GeV and $v_\chi = 10^5$ GeV, so $|\lambda_\phi| < 10^{-9}$. Moreover, from the constraint of λ_9 and suppose that $v_\eta = v_\rho \simeq 174$ GeV, $v_\phi = 10^{10}$ GeV and $v_\chi = 10^5$ GeV, then one get $|\lambda_\phi| < 10^{-10}$. The very tiny value of coupling of four scalars (λ_ϕ) can be explained by using the requirement of mass at TeV scale of the pseudoscalar A_5.

3. CP even scalar sector:
    - Mass of inflaton: $m_\Phi = \sqrt{2\lambda_{10}}\, v_\phi \approx 10^{11}$ GeV hence $\lambda_{10} \approx 1$ if $v_\phi \approx 10^{10}$GeV.
    - Mass of heavy scalar: $m_{H_\chi}^2 \approx 2\lambda_1 v_\chi^2 + \frac{\lambda_5^2}{2\lambda_1} v_\rho^2$.

- Two light scalar fields: Using the approximation $\lambda_2 \simeq \lambda_3 \simeq \lambda_6$, one have:

$$m^2_{h,h5} \approx \lambda_3 v^2 + \frac{m^2_{A_5}}{2} \pm \sqrt{m^4_{A_5} + \lambda_3^2\left(v^4 - 3v_\eta^2 v_\rho^2\right) - \frac{\lambda_3 m^2_{A_5}\left(v^4 - 2v_\eta^2 v_\rho^2\right)}{v^2}} \quad (2.25)$$

  In case of $v_\eta = v_\rho = \frac{v}{\sqrt{2}}$, then: $m_h^2 \simeq \frac{3}{2}\lambda_3 v^2\,,\qquad m^2_{h_5} \simeq \frac{\lambda_3 v^2}{2} + m^2_{A_5}$.

The h field is SMLHB. A new massive scalar h_5 with mass could be 150 GeV or 96 GeV which depends on parameters $\lambda_2, \lambda_3, \lambda_\phi$ and VEVs of scalar fields. The corellation between masses of A_5, h and h_5 as $|m^2_{h_5} - m^2_{A_5}| = \mathcal{O}\left(m_h^2\right)$.

2.9. Yukawa interactions and the flavour conservation for SMLHB's interaction

With $\alpha = 1, 2$ and $a = \alpha, 3$, the Yukawa couplings are defined by:

$$(y_6)_{na} = \frac{\sqrt{2}}{v_\rho}\left(V_{uL}\widetilde{M}_u V^\dagger_{uR}\right)_{na}, \qquad (y_3)_{3a} = \frac{\sqrt{2}}{v_\eta}\left(V_{uL}\widetilde{M}_u V^\dagger_{uR}\right)_{3a} \quad (2.26)$$

$$(y_4)_{na} = \frac{\sqrt{2}}{v_\eta}\left(V_{dL}\widetilde{M}_d V^\dagger_{dR}\right)_{na}, \qquad (y_5)_{3a} = \frac{\sqrt{2}}{v_\rho}\left(V_{dL}\widetilde{M}_d V^\dagger_{dR}\right)_{3a}. \quad (2.27)$$

The very tiny coupling λ_ϕ might be understood as the result of $U(1)_L$ symmetry breaking. This global of lepton number symmetry is violated by the interaction of four scalars $\lambda_\phi \eta\chi\rho\phi$. The Yukawa couplings can be rewritten in another form as below:

$$\left(\Gamma^h_{u,d}\right)_{ij} = \frac{c_{\alpha_2}}{v_\rho}\left(\widetilde{M}_{u,d}\right)_{ij} - \frac{c_{\alpha_2}}{v_\eta}(\tan\alpha + \tan\alpha_2)\left(\Gamma'^{(u,d)}_h\right)_{ij}. \quad (2.28)$$

In the equation (2.28), the first term is flavour conserving and the second term is a flavour changing. In order to get the flavour conservation of SMLHB's interactions, the second term should be vanished. Then, one get the condition below: $\tan\alpha = -\tan\alpha_2$..

2.10. Conclusions of chapter 2

The reason of inposing $Z_{11} \otimes Z_2$ into the $\nu 331$ model is presented in detail. The mass mixing matrices of Higgs scalar sector in the ALP331 model are diagonalized to define the physical states and masses of scalar fields which appear in ALP331 model (ALP, SMLHB,...). Gauge boson sector is also diagonalized clearly and shows that there are 5 new heavy gauge bosons $(Z', X^0, X^{0*}, Y^\pm)$ with 4 SM gauge bosons $(A, Z, W^\pm)$.

Chapter 3. Higgs sector phenomenology in the ALP331 model

3.1. Rare *top* decays by FCNCs

3.1.1. Rare decays of top *quark* $t \to ch$ *and* $t \to uh$ *by FCNC*

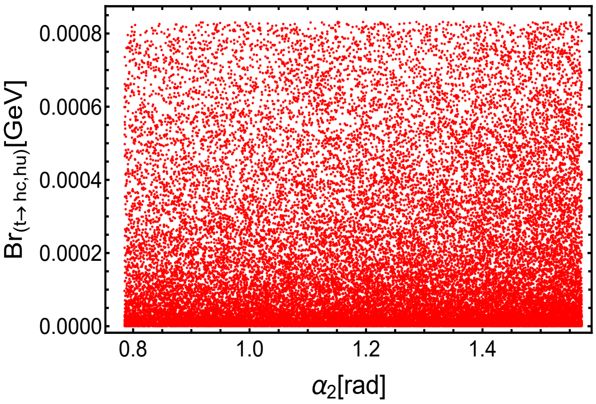

Figure 3.1: Correlation between the mixing angle α_2 and branching ratio $Br_{(t\to hu, t\to hc)}$.

The branching ratio of $t \to q h_5$ $(q = c, u)$ processes are:

$$Br(t \to h_5 u) = \frac{\frac{g^2_{th_5u}}{4\pi}\frac{(m_t^2 - m_{h_5}^2)^2}{2m_t m_{h_5}}}{\Gamma_t}, \qquad Br(t \to h_5 c) = \frac{\frac{g^2_{th_5c}}{4\pi}\frac{(m_t^2 - m_{h_5}^2)^2}{2m_t m_{h_5}}}{\Gamma_t}. \tag{3.1}$$

If mass of h_5 is about 150 GeV, the numerical analysis showed that the branching ratios of $t \to h_5 q$ (with $q = u, c$) might get the value about 10^{-3}.

3.1.2. Rare decays of top *quark* $t \to c\gamma$ *and* $t \to u\gamma$ *by FCNC*

The width decays of these processes $t \to c\gamma$ and $t \to u\gamma$ have forms:

$$\begin{aligned}
\Gamma(t \to c\gamma) &= \frac{\alpha G_F m_t^3 \left|y_{hct}\right|^2}{192\pi^4}\left|\left(f_1\frac{m_h}{m_t} + f_2\frac{m_h}{m_t}\right)A_h B_h + \left(f_1\frac{m_{h_5}}{m_t} + f_2\frac{m_{h_5}}{m_t}\right)A_{h_5}B_{h_5}\right|^2 \\
\Gamma(t \to u\gamma) &= \frac{\alpha G_F m_t^3 \left|y_{hut}\right|^2}{192\pi^4}\left|\left(f_1\frac{m_h}{m_t} + f_2\frac{m_h}{m_t}\right)A_h B_h + \left(f_1\frac{m_{h_5}}{m_t} + f_2\frac{m_{h_5}}{m_t}\right)A_{h_5}B_{h_5}\right|^2
\end{aligned} \tag{3.2}$$

If mass of the A_5 field is very near TeV scale while mass of the h_5 field is about 90 GeV$\leq m_{h_5} \leq$200 GeV then the leading contribution of $t \to u\gamma$ and $t \to c\gamma$ decays will arise from the virtual exchange of the *top* quark and neutral CP even scalar h, h_5, being h the 125GeV SMLHB. The Yukawa couplings are in range of 10^{-2} GeV$\leq y_{hct}, y_{hut} \leq 1.2 \times 10^{-2}$. The branching ratios of rare decays $t \to c\gamma$ and $t \to u\gamma$ are:

$$Br\left(t \to c\gamma\right) = \frac{\Gamma\left(t \to c\gamma\right)}{\Gamma_{top}}, \qquad Br\left(t \to u\gamma\right) = \frac{\Gamma\left(t \to u\gamma\right)}{\Gamma_{top}}, \tag{3.3}$$

where, $\Gamma_{top} = 1.42^{+0.19}_{-0.15}$ GeV is total decay width *top* quark. Numerical analysis of branching ratios of $t \to c\gamma$ and $t \to u\gamma$ shows that these ratios should be about 10^{-10}, some orders smaller than the upper limit of experiments 2.2×10^{-4} and 6.1×10^{-5}.

3.2. Decays of SMLHB h into two fermions

3.2.1. SMLHB decays into two d quarks: $h \to \bar{b}b$

Use the equation (1.4), one can define the decay width of the process $h \to \bar{b}b$. The deviation factor from the SMLHB and b quark coupling $g_{hb\bar{b}}$ in the ALP331 model and $g^{SM}_{hb\bar{b}}$ of SM is:

$$a_{hb\bar{b}} = 2\frac{c_{\alpha_2}}{c_\alpha}\,. \tag{3.4}$$

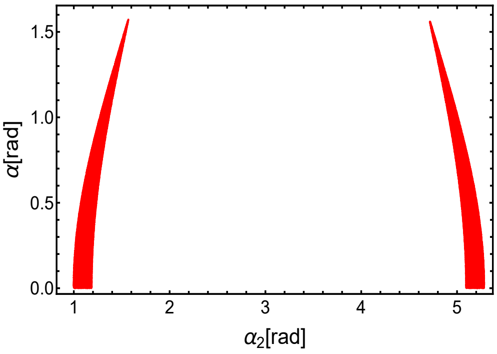

Figure 3.2: Corellation between mixing angles α and α_2 when considering the $h \to \bar{b}b$ decay.

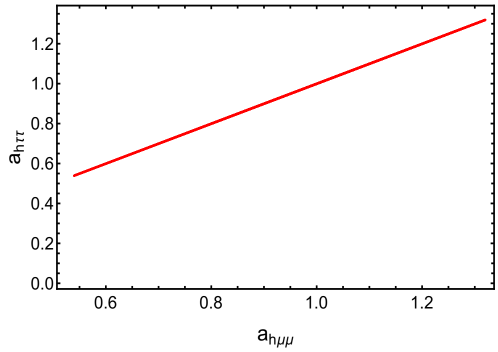

Figure 3.3: Correlation between $a_{h\tau\tau}$ and $a_{h\mu\mu}$ parameters.

To get the consistent branching ratios of $h \to \bar{b}b$ with $\alpha \in [0, \frac{\pi}{2}]$ rad, the values of α_2 should be in range of $(1 \div 1.6)$ rad or $(4.75 \div 5.4)$ rad.

3.2.2. SMLHB decays into two charged leptons $h \to \bar{l}l$

The deviation factor from $g_{h\bar{l}l}$ of ALP331 model with $g^{SM}_{h\bar{l}l}$ of SM is $a_{h\bar{l}l} = \frac{v c_{\alpha_2}}{v_\rho}$. The set of this deviation factor is $0.6 < a_{h\mu\mu,\tau\tau} < 1.2$ which is consistent with the experimental limit. Figure (3.3) describes the linear correlation between $a_{h\tau\tau}$ and $a_{h\mu\mu}$. Combine the constraint of mixing angles α and α_2 which got from the decay $h \to \bar{b}b$, one get narrow constraint of α and α_2: If 0.7 rad $\leqslant \alpha \leqslant 1.4$ rad then 1 rad $\leqslant \alpha_2 \leqslant 1.4$ rad, or 4.8 rad $\leqslant \alpha_2 \leqslant 5.4$ rad.

3.3. Meson oscillations

The meson oscillations are caused by interactions of scalar and Z' boson with quarks in the d type quark sector. The $K^0 - \bar{K}^0$, $B^0_d - \bar{B}^0_d$ and $B^0_s - \bar{B}^0_s$ meson mixings are described by effective Hamiltonians. In the ALP331 model, the new physics contributions to the meson differences depend on parameters $G_F, f_K, B_K, \eta_K, m_K,$...

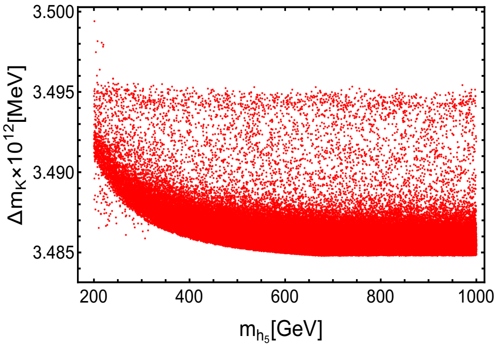

Figure 3.4: Corellation between the meson mass spliting Δm_K with mass of new light scalar m_{h_5}.

In the Figure (3.4), we plot the corellation between mass spliting of mesons Δm_K and mass of the non-SM CP even scalar m_{h_5}. In the Figure (**??**), we present the consistent region with mass spliting of mesons Δm_K, Δm_{B_d} and Δm_{B_s} constraints in the $m_{A_5} - m_{h_5}$ plane. This allowed region is in range of experimental data.

3.4. Interactions of SMLHB h with pseudoscalars in CP odd scalar sector

The coupling of SMLHB h with two ALP a is defined by:

$$g_{haa} \approx \frac{v_\rho v_\eta}{2\sqrt{2}} \left(\frac{\lambda_6 \lambda_{12}}{\sqrt{V_{236}^2 + (\lambda_3 v_\rho^2 - \lambda_2 v_\eta^2) V_{236}}} - \lambda_{13} \sqrt{V_{236} + \lambda_3 v_\rho^2 - \lambda_2 v_\eta^2} \right), \tag{3.5}$$

where, $V_{236} = \sqrt{\left(\lambda_2 v_\eta^2 - \lambda_3 v_\rho^2\right)^2 + \lambda_6^2 v_\eta^2 v_\rho^2}$.
We also get the coupling of SMLHB h with two pseudoscalars A_5 is:

$$\begin{aligned} g_{hA_5A_5} \quad &\approx \quad \frac{1}{2\sqrt{2}} \left(v_\rho (2\lambda_3 v_\eta^2 + \lambda_6 v_\rho^2) \sqrt{\frac{V_{236} - \lambda_3 v_\rho^2 + \lambda_2 v_\eta^2}{V_{236}}} \right. \\ &\qquad \left. - v_\eta (2\lambda_2 v_\rho^2 + \lambda_6 v_\eta^2) \sqrt{\frac{V_{236} + \lambda_3 v_\rho^2 - \lambda_2 v_\eta^2}{V_{236}}} \right). \end{aligned} \tag{3.6}$$

Considering the new light Higgs boson h_5, we recognize that the couplings g_{h_5aa} and $g_{h_5A_5A_5}$ also depend on $\lambda_2, \lambda_3, \lambda_6, v_\rho, v_\eta$ and V_{236}.

3.5. Conclusions of chapter 3

Phenomenology in ALP331 such as meson ocsillation, rare *top* quark decays, and the decay of SMLHB into 2 leptons caused by FCNC are studied to limit the bounds of some parameters in the model (mixing angles, interactions' couplings,...).

CONCLUSIONS

The ALP331 has two types of scalar fields which are bilepton scalars carry lepton number 2 and ordinary scalars carry zero of lepton number. The physical fields of a and pseudoscalar A_5 are defined exactly. These two fields interact with fermions in ALP331 with very tiny strength.

Diagonalizing the mass mixing matrices to define the physical fields is the initial step to study on the phenomenology of ALP331.

Numerical analysis of scalar sector gives us the lower limit of mass of pseudoscalar A_5 which is near 200GeV. This limit is completely diffirent from the other results published before. We also point out the constraints of the couplings $\lambda_2, \lambda_3, \lambda_\phi$ with $\tan\alpha = \frac{v_\eta}{v_\rho}$ and VEVs of scalar fields ϕ, χ, η, ρ to arise a new light Higgs boson with mass at TeV or subTeV scale.

The ALP331 model is consistent with the experimental constraints of rare *top* decays by FCNCs, SMLHB decays as well as meson oscillations $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$.

The ALP331 model can explain either the number of fermions' generation or the very tiny mass of neutrino. It also includes the DM candidate and the very heavy mass of inflaton in the Early Universe, The appearence of ALP can not solve the SCPp but ALP can be another type of DM (not axion QCD DM). All these results are natural. Hence, keeping this research direction to solve the problems beyond SM is feasible and necessary.

NEW CONTRIBUTIONS

1. We analized the spectrum of particles of the model in detail to point our the missing and incorrect conclusions about some new particles in the scalar sector which were published by the other authors. Then we add the missing terms and correct the mass mixing matrices. These matrices are diagonalized to define the physical states and masses of fields in scalar sector of ALP331 model.
2. Physical states of scalar fields shows that the model under consideration does not include the axion which is a candidate of CDM - QCD axion. There is only ALP in the model so that the Strong CP problem can not be solved. ALP is not raised from QCD interactions but ALP is still a candidate of diffirent type DM. For these reason, the $\nu 331$ imposing $Z_{11} \otimes Z_2$ discrete symmetry or A331 model is renamed as ALP331 model.
3. Phenomenology which have not been studied in the ALP331 before, are studied by us. Those are meson ocsillation, the flavour conservation of SMLHB decay, the FCNCs cause rare *top* quark decays $t \to hq, t \to q\gamma$ with $q = u, c$, interactions between scalar field with exotic quarks and new interactions of new particles h, h_5, a, A_5. These results helps us find out the constraints of some parameters such as mass mixing angles, interaction's couplings inf the ALP331 to fit with data from experiments.

PUBLICATIONS

1. D. T. Binh, L. T. Hue, V. H. Binh, H. N. Long, *Neutrino energy-loss rate in* 331β *model*, International Journal of Modern Physics A, Vol **36**, No. 26 (2021) 2150179.
2. Hoang Ngoc Long, Vo Van Vien, Vu Hoa Binh, *Effective Couplings between Two Photons and Axion in the 3-3-1 Model*, VNU Journal of Science: Mathematics - Physics, Vol. **37**, No. 4 (2021), 51-58.
3. V. H. Binh, D. T. Binh, A. E. Cárcamo Hernández, D. T. Huong, D. V. Soa, and H. N. Long, *Higgs sector phenomenology in the 3-3-1 model with an axionlike particle*, arXiv:2007.05004[hep-ph], Phys. Rev. D **107**, 095030 (2023), (28 pages), DOI: 10.1103/PhysRevD.107.095030, ISSN: 1550-7998, IF: 5.407 (SCI), h-5 index: 155.
4. L. T. Hue, H. N. Long, V. H. Binh, H. L. T. Mai, T. Phong Nguyen, *One-loop contributions to decays* $e_b \to e_a\gamma$ *and* $(g-2)_{e_a}$ *anomalies and Ward identity* , arXiv:2301.05407 [hep-ph], Nuclear Physics B **992**, (2023), 116244. DOI: 10.1016/j.nuclphysb.2023.116244, IF: 3.045 (SCI), h index: 262.
5. V. H. Binh, D. T. Huong, H. N. Long, D. V. Soa, *New scalar fields in the 3-3-1 model with axion like particle*, The 48th Vietnam Conference on Theoretical Physics, (2023).

The third publication is used in the thesis. This thesis relates to the second publication and the report at the 48th Vietnam Conference on Theoretical Physics.

Bibliography

[1] J. C. Montero, V. Pleitez and O. Ravinez, *Soft superweak CP violation in a 3-3-1 model*, Physics Review D, **1999**, 60, 076003, [hep-ph/9811280].

[2] J. C. Montero, C. C. Nishi, V. Pleitez, O. Ravinez and M. C. Rodriguez, *Soft CP violation in K-meson systems*, Physics Review D, **2006**, 73, 016003 [hep-ph/0511100].

[3] V. H. Binh, D. T. Binh, A. E. Cárcamo Hernández, D. T. Huong, D. V. Soa, and H. N. Long, *Higgs sector phenomenology in the 3-3-1 model with axion like particle*, Physics Review D, **2023**, 107, 095030.

[4] A. G. Dias, C. A. de S. Pires and P. S. Rodrigues da Silva, *Discrete symmetries, invisible axion and lepton number symmetry in an economic 3-3-1 model*, Physics Review D **2003**, 68, 115009.

[5] J. G. Ferreira, C. A. de S. Pires, J. G. Rodrigues and P.S. Rodrigues da Silva, *Embedding cosmological inflation, axion dark matter and seesaw mechanism in a 3-3-1 gauge model*, Physics Letter B **2017**, 771, 199.

[6] S. Weinberg, *A New Light Boson?*, Physics Letter B **1978**, 40, 223.

[7] F. Wilczek, *Problem of Strong P and T Invariance in the Presence of Instantons*, Physics Review Letter **1978**, 40, 279.

[8] I. I. Bigi, A. I. Sanda, *CP violation*, Cambridge University Press, **2009** https://doi.org/10.1017/CBO9780511581014.

[9] C. B. Adams *et al., Snowmass 2021 White Paper Axion Dark Matter*, **2023**, https://doi.org/10.48550/arXiv.2203.14923.

DOI: 10.1140/epjc/s10052-023-11203-9, arXiv: 2206.10100 [hep-ph].