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THE PROBLEM OF PIEZOELECTRIC ENERGY HARVESTING OF NONLINEAR CANTILEVER BEAM MODEL IN THE RESONANCE DOMAIN INCLUDING PRIMARY AND SECONDARY RESONANCES

SUMMARY OF DISSERTATION
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INTRODUCTION

In previous studies, most of the works focused on developing energy-harvesting devices based on linear resonance effects. However, the efficiency of the linear piezoelectric energy harvesting system (PEHs) is limited to a very narrow band around the resonant frequency. Any deviation of the excitation frequency away from the resonance vicinity can lead to a drastic reduction in the amount of recovered power. Among the analytical methods, averaging method is one of the most effective and powerful techniques to analyze nonlinear phenomena in dynamic systems. Although the averaging method has long been used in mechanics. However, to the best of the Ph.D. student's knowledge, there is no published research to determine the analytic expressions of the device electromechanical response with a nonlinear one-degree-of-freedom model, under harmonic agitation, in different resonance effects when using averaging method. Developing theoretical research methods to analyze nonlinear electromechanical system responses, energy collection efficiency evaluation becomes a topic of scientific and practical significance.

The objective of this thesis is to develop an average method for a nonlinear electromechanical system subjected to harmonic excitation with lumped mass model a one-degree-of-freedom of device harvests energy piezoelectric; from that result, applied to the mono-stable Duffing PEHs at nonlinear resonance effects including main, secondary resonance and linear respectively used to compare; The analy, surveys, and evaluate the influence of parameters on the amplitude-frequency relationship, the responses of the nonlinear electromechanical system, and the corresponding linear systems used for comparison.

Research object of the present thesis is a mono-stable Duffing PEHs subjected to harmonic excitation with lumped mass model a single-degree-of-freedom of device harvests energy piezoelectric.

Scope of the present thesis studies the device harvests energy piezoelectric with a cantilever beam structure, beams fitted with piezoelectric layer on the entire upper and lower surfaces, cantilever beams have no mass added at the free end, the research beam has a rectangular cross-section, the cantilever beam structure with two piezoelectric layers is subjected to harmonic excitation at the section adjacent to the mount end and on the basis of the Euler–Bernoulli beam theory, considering to the nonlinear strain-displacement relations. The structure of the research device can be simplified by considering only the first vibration mode is modeled as a lumped mass model a single-degree-of-freedom subjected to harmonic excitation.

Methodology used in this study is mainly the average method and combines the analyzed, surveyed, and evaluated by numerical accomplished in MATLAB code.

Content of thesis consists of followings:

Chapter 1 presents an overview of energy harvesting using piezoelectric materials and introduces the research content of the thesis.

Chapter 2 presents the construction, setting, and determination of the electromechanical system of connection equations of the beam structure with two layers of piezoelectric materials, and the first vibration mode is modeled as a lumped mass model a single-degree of freedom of the mono-stable Duffing-type subjected to a harmonic base excitation.

Chapter 3 presents the content of developing the average method used for the mono-stable Duffing PEHs at nonlinears resonance effects different. Determining the expressions of the response of the electromechanical system the thesis studied the resonance effects with the nonlinear and linear systems respectively.

Chapter 4 presents the analysis and evaluation of the influence of parameters on the responses of the electromechanical system the thesis studies in nonlinear resonance effects including primary and secondary resonance, and corresponding linearity for comparison using the Matlab software tool.

Conclusion presents main results obtained in the thesis and subject of further study for the author.

Novelty of results obtained in this thesis can be formulated as:

1. On the basis of modeling a set of nonlinear piezoelectric energy harvesters with a cantilever beam structure attached with piezoelectric layers by a single degree of freedom model with a lumped mass subject to harmonic, the thesis has established the system of differential equations for the Duffing nonlinear oscillations of the one-degree-of-freedom electromechanical system;

2. The thesis has developed the averaging method for a nonlinear electromechanical system subjected to harmonic excitation with lumped mass model a single-degree-of-freedom of piezoelectric energy harvester; from that result, applied to the mono-stable Duffing PEH system in the resonance domain including primary and secondary resonances and linear respectively used to compare;
3. The thesis has used the content developed, and expanded the average method to determine the analytic expressions of the relationship between amplitude-frequency, displacement response, voltage, input and output mechanical power, mechanical and useful electrical energy, and efficiency of energy harvesting in nonlinear resonance and linear respectively used to compare;
4. The thesis has analyzed, surveyed, and evaluated the influence of parameters on displacement responses, voltage, input mechanical power, output, input mechanical energy, output potential useful electrical energy, and efficiency of energy harvesting of the electromechanical system the thesis studied in nonlinear resonance effects including primary, secondary resonance, and linear respectively used to compare;
5. The thesis has survey results showing that: Amplitude and frequency of excitation are two parameters that greatly affect the responses of the electromechanical system; The frequency range near the resonance domain of the nonlinear resonant of the nonlinear PEHs is wider than the corresponding linear system; The efficiency energy harvests of the electromechanical system are larger in the resonance domain of subharmonic resonance but smaller in the resonance domain of superharmonic and primary resonance. However, the input-output mechanical power, input mechanical energy, and use of electrical energy in the resonance effects are always larger than the corresponding linear system;

Chapter 1. OVERVIEW PIEZOELECTRIC ENERGY HARVESTING

1.1. Contents about piezoelectric energy harvesting

Energy harvesting is defined as the direct conversion of energy from the surrounding environment into electricity by using a transitional material or mechanism (the surrounding environment including mechanical, solar, heat, wind, liquid flow, etc.)... According to Williams and Yates, there are three mechanisms to energy harvesting from vibration to electricity: electromagnetic, electrostatic, and piezoelectric; The energy harvesting of mechanical vibrational energy into electricity using piezoelectric materials is called piezoelectric energy harvesting. According to aggregated data by Ghazanfarian et al., in the last two decades excluding conference reports and assessments, the keyword "piezo and energy harvesting" extracted from Scopus is shown in *Fig 1.1*. This result shows the attraction, a significant increase in demand, and application of piezoelectric materials as well as research trends.

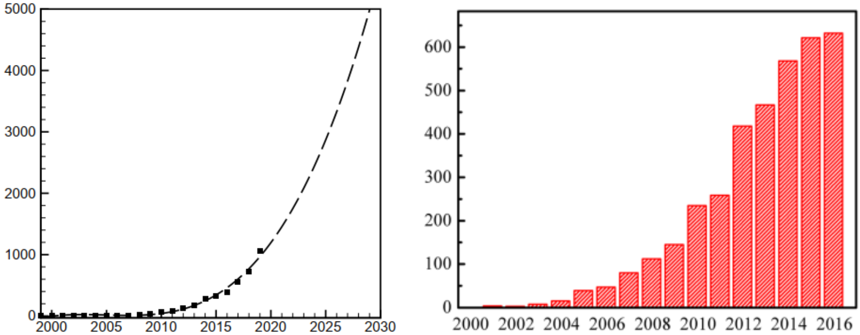

Fig 1.1. Overall history and future estimation of publications on piezoelectric energy harvesting

The relationship of stress, strain, electric field intensity, and electric displacement of the form (3-1) is shown:

$$\begin{aligned} \sigma_3 &= c_{33}S_3 - e_{31}E_3 \\ D_3 &= e_{31}S_3 + \varepsilon_{33}E_3 \end{aligned} \tag{1.2}$$

The system of equations (1.2) is the basis of the linkage equations in the electromechanical system for piezoelectric energy harvesting studied and used in this thesis.

1.2. Structure and mathematical modeling of piezoelectric energy harvesters

According Li et al, *Figure 1.4* shows the papers' number on typical structure of PEHs in the database of Web of Science using cantilever piezoelectric energy harvesting, cymbal piezoelectric energy harvesting, and stack piezoelectric energy harvesting as key words, respectively. Results show that researchers pay much attention on the cantilever PEHs as the simple fabrication process and relatively larger strain.*Fig 1.5*. Nonlinearity in typical designs of piezoelectric energy collectors with cantilever beam structure has been announced by popular scientists in two aspects that are: Nonlinearity of beam structure (mainly based on the large deformation properties of the basic beam structure) and the nonlinear properties of the piezoelectric layer mounted on the beam (a substrate layer).

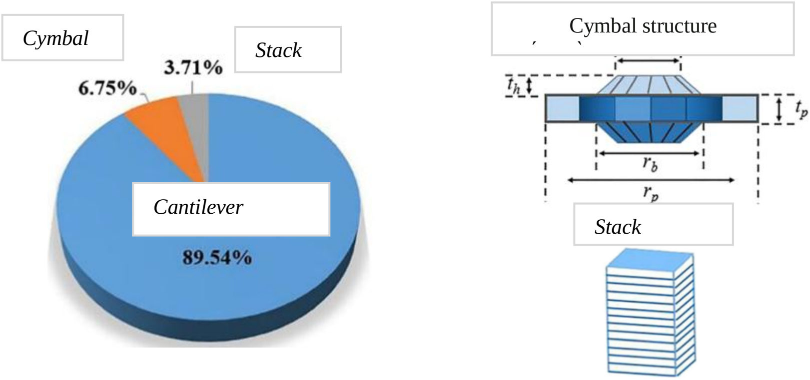

Fig 1.4. Published papers' number on the typical structure of PEHs from 2000 to 2016

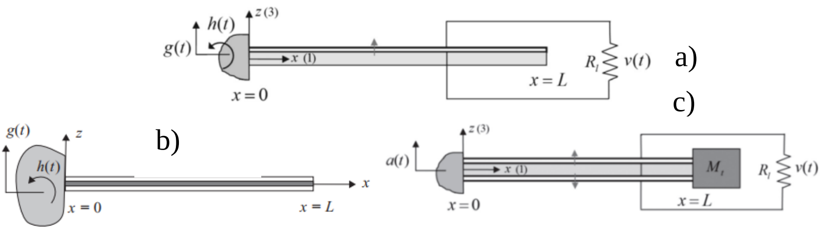

Fig 1.5. Geometrically piezoelectric energy harvester configurations with a) uniform unimorph b) uniform bimorph c) uniform bimorph with tip mass

1.3. Research orientation and problem formulation

Main research content in this thesis:

Firstly, setting up a system of differential equations describing a set of energy-collecting devices with a cantilever beam structure subjected to the agitation of the air conditioning platform, the beam with a piezoelectric layer on the entire upper and lower surface of the beam, cantilever beam without tip mass, the studied beam has a rectangular cross-section, based on the Euler–Bernoulli beam theory, considering the nonlinear relationship of displacement and strain. Since then, the thesis has modeled the structure cantilever beam with a piezoelectric layer attached in the first mode shape of vibration of the piezoelectric energy harvester by a lumped mass model of one degree of freedom subjected to a harmonic ground motion (Chapter 2);

Second, develop averaging method for Duffing type nonlinear electromechanical system, subjected to harmonic base excitation with a one-degree-of-freedom lumped mass model of piezoelectric energy harvester, from which applied to the single-well system in nonlinear resonance effects including superharmonic, sub resonance, main resonance, and linear system for comparison with the corresponding nonlinear system, the expressions amplitude-frequency relationship analysis, displacement responses, voltage, input and output mechanical power, mechanical energy, useful electrical energy, and efficiency piezoelectric energy harvesting are determination (Chapter 3);

Third, use the Matlab program to survey and evaluate the influence of parameters on displacement response, voltage, input mechanical power, output, input mechanical energy, useful electrical energy, output potential, efficiency piezoelectric energy harvesting the electromechanical system studied in the effects related to resonance phenomena including main resonance, secondary resonance and corresponding linearity (Chapter 4);

Chapter 2. SETTING OF GOVERNING EQUATIONS FOR CANTILEVER BEAM NONLINEAR PIEZOELECTRIC ENERGY HARVESTERS

2.1. The system of electromechanically coupled governing equations for geometric nonlinearity cantilever beam piezoelectric energy harvesters

Consider a bimorph piezoelectric energy harvester is shown in *Fig 2.1* and with cross-section $A_1 - A_1$ *(Fig 2.1.b.)* Based on Euler–Bernoulli beam theory, the axial displacement u_1 and the transverse displacement w_1, at any point of the element are given by

$$\begin{cases} u_1 = -z_1 \dfrac{\partial w_0}{\partial x_1} \\ v_1 = 0 \\ w_1 = w_0(x_1) \end{cases} \tag{2.1}$$

where, (u_1, v_1, w_1) are respectively the displacements in the directions of the axis (x_1, y_1, z_1), respectively, and w_0 is the deflection at any point on the neutral axis and z_1 is the distance from the considering point to the mid-plane.

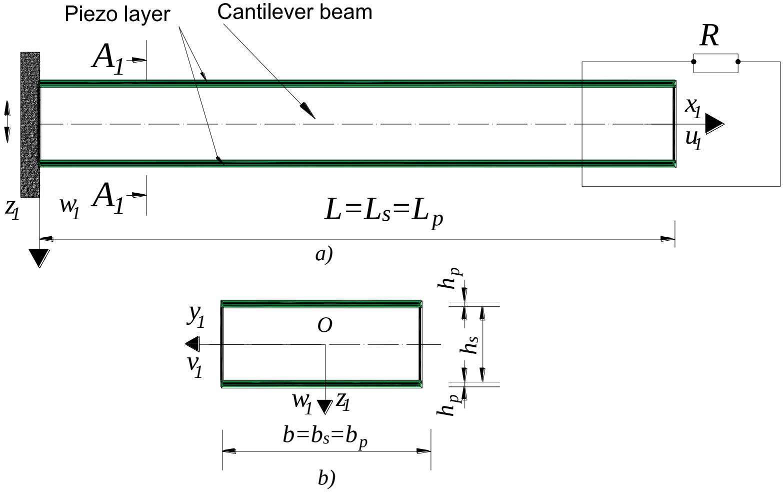

Fig 2.1. Schematic diagram of piezoelectric bimorph cantilever beams; b) The cross section of the piezoelectric cantilever beam

A degenerated form of Green's strain resulted from the displacements can be adopted for the local axial strain $\varepsilon_{x-s}(x_1, z_1, t)$, can be as

$$\varepsilon_{x-s}(x_1, z_1, t) = \varepsilon_{x0}(x_1, t) - z_1 \frac{\partial^2 w_1}{\partial x_1^{\ 2}} \tag{2.3}$$

Therefore, the nonlinear displacement at the point considering the neutral axis ($z_1 \neq 0$)

$$\varepsilon_{x-s} = \varepsilon_{x0} - z_1 \frac{\partial^2 w_1}{\partial x_1^{\ 2}} = \frac{1}{2}\left(\frac{\partial w_1}{\partial x_1}\right)^2 - z_1 \frac{\partial^2 w_1}{\partial x_1^{\ 2}} \tag{2.4}$$

$$\varepsilon_{x-p} = \varepsilon_{x-s} \tag{2.5}$$

Assuming the linearly elastic behavior, the axial stress-strain, and the axial stress - electric displacement, the electric field- axial strain of the substructure and the layers of the piezoelectric material, respectively as

$$\sigma_x = E_s \varepsilon_{x\_s} \tag{2.6}$$

$$\sigma_{x-p} = E_p\varepsilon_{x-p} - e_{31}E_3$$
$$D_p = e_{31}\varepsilon_{x-p} + \varepsilon_{33}E_3; \qquad (2.7)$$

where, $\sigma_x;\sigma_{x-p}$ the are the stress components (in the x-direction) in the substructure and the piezoelectric layers; the axial strain in the substructure and the piezoelectric layers are defined as ε_{x-s} and ε_{x-p}, respectively. $E_s;E_p$ are the elastic modulus of the substructure and the piezoelectric layers, e_{31} is the effective piezoelectric stress constant, ε_{33} is the permittivity component at constant strain with the plane-stress assumption for a beam; D_p is the electric displacement. E_3 is the electric field component in the 3 direction. v_p is the voltage across the electrodes of each piezoceramic layer can be expressed as :

$$E_3 = -\frac{v_p}{2h_p} \qquad (2.8)$$

The potential energy per unit volume of beam bimorph PEH

$$\bar{W} = \frac{1}{2}E_s\varepsilon_{x-s}^2 + 2\left(\frac{1}{2}E_p\varepsilon_{x-p}^2 + e_{31}\varepsilon_{x-p}\frac{v_p}{2h_p} - \varepsilon_{33}\frac{v_p^2}{8h_p^2}\right) \qquad (2.11)$$

The potential energy for an element with initial length of L reads

$$W_{el} = \int_0^L \frac{1}{2}E_bA_b\left[\frac{1}{2}\left(\frac{\partial w_1}{\partial x_1}\right)^2\right]^2 dx_1 + \int_0^L \frac{1}{2}I_bE_b\left(\frac{\partial^2 w_1}{\partial x_1^{\,2}}\right)^2 dx_1 -$$
$$-\int_0^{L_p} \vartheta\varepsilon_{33}\frac{v_p^2(t)}{4h_p^2}dx_1 + \int_0^{L_p}\vartheta\left(\frac{\partial w_1}{\partial x_1}\right)^2 v_p(t)dx_1 \qquad (2.19)$$

The kinetic energy for an element with initial length real L

$$T_s = \frac{1}{2}\int_0^{L_s}\left(\frac{\partial w_1}{\partial t} + \frac{\partial z_1}{\partial t}\right)^2 \rho_sA_s(x)dx_1; \qquad (2.20)$$

$$T_p = \frac{1}{2}\int_0^{L_p}\left(\frac{\partial w_1}{\partial t} + \frac{\partial z_1}{\partial t}\right)^2 \rho_pA_p(x)dx_1;$$

(2.21)

Assuming that the influence of gravity is ignored, and the imaginary work of the charge displacement, then the imaginary work of the resistance is determined by

$$W_{nc}^{m} = -\int_{t_1}^{t_2}\int_{0}^{L} c_t\left(\frac{\partial w_1(x_1,t)}{\partial t}\right)^2 dx_1 dt \tag{2.23}$$

Applying Hamilton's principle, Hence S defined as

$$S = \int_{t_1}^{t_2}(W_{el} - T_s - T_p + W_{nc}^{m})dt \tag{2.24}$$

the Euler–Lagrange equation of motion of the Euler–Bernoulli beam as:

$$\begin{gathered} m_b\frac{\partial^2 w_1}{\partial t^2} + c_b\frac{\partial w_1}{\partial t} + E_b I_b\frac{\partial^4 w_1}{\partial x^4} - E_b A_b\frac{\partial}{\partial x_1}\left\{\frac{\partial w_1}{\partial x_1}\frac{1}{2}\left(\frac{\partial w_1}{\partial x_1}\right)^2\right\} - \\ -4\vartheta v_p(t)\left[\frac{d\delta(x_1)}{dx_1} - \frac{d\delta(x_1 - L_p)}{dx_1}\right] = -m_b\frac{\partial^2 z_1}{\partial t^2}; \end{gathered} \tag{2.30}$$

Equation (2.30) is the partial differential equation describing the bending vibrations of a bimorph cantilever beam piezoelectric energy harvester, the term component $\frac{E_b A_b}{2L}\frac{\partial^2 w_1}{\partial x_1^2}\left[\int_0^L\left(\frac{\partial w_1}{\partial x_1}\right)^2 dx_1\right]$ is representing the nonlinearity between displacement and strain when considering the very small high-order strain. Then, Kirchhoff’s laws, one may obtain the governing equation of electrical circuit of the system, as follows:

$$C_p\frac{dv(t)}{dt} + \frac{v(t)}{R} = -\vartheta\int_{x_1=0}^{L_p}\left\{\frac{\partial^3 w_1}{\partial x_1^2 \partial t}\right\}dx_1; \tag{2.37}$$

In order to solve Eqs.(2.30) and (2.37), the Galerkin method is utilized to discretize the partial differential equations. These conditions are, respectively, given by:

$$x_1 = 0: \qquad w_1(0,t) = 0; \frac{\partial w_1}{\partial x_1}(0,t) = 0; \tag{2.38}$$

$$x_1 = L: \qquad \frac{\partial^2 w_1}{\partial x_1^2}(L,t) = 0; \frac{\partial^3 w_1}{\partial x_1^3}(L,t) = 0 \tag{2.39}$$

The transversal displacement $w_1(x_1,t)$ can be assumed as

$$w_1(x_1,t) = X_1(x_1)h_1(t) \tag{2.40}$$

The mode shapes are calculated as:

$$X_1(x_1) = \left[\cosh\frac{\lambda_1 x_1}{L} - \cos\frac{\lambda_1 x_1}{L} - \lambda^*\left(\sinh\frac{\lambda_1 x_1}{L} - \sin\frac{\lambda_1 x_1}{L}\right)\right] \quad (2.41)$$

where the relation between $\beta_1 = \frac{\lambda_1}{L}$ and ω_1 is given by $\omega_1 = \beta_1^2\sqrt{\frac{E_b I_b}{m_b}}$

By substituting equation $w_1(x_1,t) = X_1(x_1)h_1(t)$ into Eqs. (2.51) and (2.57), multiplying the resulting equations by $X_1(x_1)$, and integrating over the length of the beam, the following nonlinear ordinary differential equations can be obtained

$$\begin{cases} M_1\ddot{h}_1(t) + c_f\dot{h}_1(t) + K_1h_1(t) + K_3h_1^3(t) + \chi v_p(t) = -M_2\ddot{z}_1(t) \\ C_p\frac{dv_p(t)}{dt} + \frac{v_p(t)}{R} = \theta\dot{h}_1(t) \end{cases} \quad (2.53)$$

2.2. Modeling of nonlinear cantilever beam bimorph piezoelectric energy harvesters

Consider the lumped-parameter model of a cantilever piezoelectric energy harvesting (PEH) system subjected to base excitation is illustrated in *Fig. 2.3*.

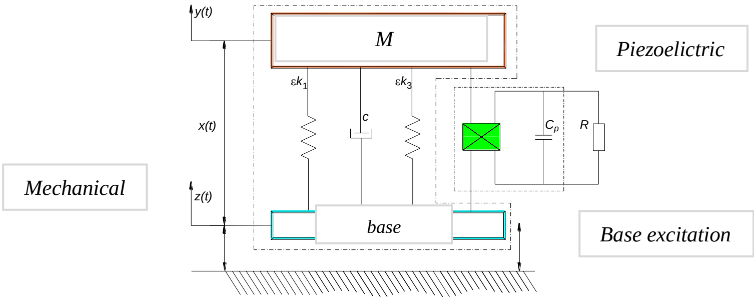

Fig 2.3. Schematic of a PEH system

$$C_p\dot{V} + \frac{1}{R}V = \theta\dot{x} \quad (2.59)$$

$$M\ddot{x} + \varepsilon c\dot{x} + k_1x + \varepsilon k_3x^3 + \varepsilon\theta V = -M\ddot{z} \quad (2.60)$$

The base excitation is given in the form

$$z = \varepsilon A\cos\Omega t;\ \dot{z} = -\varepsilon A\Omega\sin\Omega t;\ \ddot{z} = -\varepsilon A\Omega^2\cos\Omega t \quad (2.61)$$

The governing equations of the system are written as follows

$$\ddot{x}+\omega_0^2 x=\varepsilon f\left(x,\dot{x},v\right)+\varepsilon A\Omega^2\cos\Omega t \qquad (2.63)$$

$$\dot{v}+\alpha v=\dot{x} \qquad (2.64)$$

where $$f\left(x,\dot{x},v\right)=-\left(2\xi\dot{x}+\gamma x^3+\kappa^2 v\right) \qquad (2.65)$$

In chapter 2, the thesis presented the following main results:

1. There was studied, establish the system of electromechanical linkage equations of a bimorph piezoelectric energy harvester nonlinear cantilever beam when considering the nonlinear relationship of displacement and deformation (geometrical nonlinearity when considering to infinity small order of deformation);
2. There was the studied beam structure has been modeled in the form of the first specific vibration by the Duffing nonlinear single degree of freedom lumped mass parameter model, subject to excitation as harmonic.

Chapter 3. DEVELOPMENT OF THE AVERAGE METHOD USED FOR A NONLINEAR ELECTROMECHANICAL SYSTEM SUBJECTED TO BASE EXCITATION WITH A SINGLE DEGREE OF FREEDOM LUMPED PARAMETER MODEL OF A CANTILEVER PIEZOELECTRIC ENERGY HARVESTER

3.2. Development of the average method used for a nonlinear electromechanical system subjected to base excitation with a single degree of freedom lumped parameter model

The Eq. (2.63) is transformed to the standard form by setting

$$x=y+f_0\cos\Omega t;\ f_0=\frac{\varepsilon A\Omega^2}{\omega_0^2-\Omega^2} \qquad (3.12)$$

The system of equations (2.63), (2.64), becomes

$$\ddot{y}+\omega_0^2 y=\varepsilon f\left(y+f_0\cos\Omega t,\dot{y}-\Omega f_0\sin\Omega t,v\right) \qquad (3.13)$$

$$\dot{v}+\alpha v=\dot{y}-\Omega f_0\sin\Omega t \qquad (3.14)$$

In nonlinear oscillator systems, resonance phenomena are observed not only when $\Omega\approx\omega_0$ one may use approximate relations between ω_0 and Ω

$$\omega_0^2 = \frac{m^2}{n^2}\Omega^2 - \varepsilon\sigma \tag{3.15}$$

where σ is a detuning parameter. According to the averaging method, the solutions $y, \dot{y}$ can be considered as the solutions of the linear equation by putting $\varepsilon = 0$ in Eq. (3.13), but constant terms are assumed as function of time. Hence one has

$$y = a(t)\cos\left[\frac{m}{n}\Omega t + \varphi(t)\right] \tag{3.16}$$

$$\dot{y} = -a(t)\frac{m}{n}\Omega\sin\left[\frac{m}{n}\Omega t + \varphi(t)\right] \tag{3.17}$$

denote:

$$\phi(t) = \frac{m}{n}\Omega t + \varphi(t) \tag{3.18}$$

Substituting Eq. (3.16), (3.17) into Eq. (3.14) gives

$$v = B\sin\phi + C\cos\phi + D\sin\Omega t + E\cos\Omega t \tag{3.19}$$

Therefore:

$$v(t) = \left\{ \begin{aligned} &-\frac{a(m/n)\Omega}{\alpha^2 + (m/n)^2\Omega^2}\left(\alpha\sin\phi - (m/n)\Omega\cos\phi\right) \\ &+\frac{f_0\Omega}{\alpha^2 + \Omega^2}\left(\Omega\cos\Omega t - \alpha\sin\Omega t\right) \end{aligned} \right\} \tag{3.21}$$

one gets the system of differential equations for $\dot{a}$ and $\dot{\varphi}$ as follows

$$\dot{a} = -\frac{\varepsilon n}{m\Omega}\left[\sigma a\cos\phi + f^{tb}\right]\sin\phi \tag{3.26}$$

$$\dot{\varphi} = -\frac{\varepsilon n}{am\Omega}\left[\sigma a\cos\phi + f^{tb}\right]\cos\phi \tag{3.27}$$

Considering a, φ to be constants during this averaging process one obtains the following averaged equations:

$$\langle\dot{a}\rangle = \frac{\varepsilon n}{m\Omega}S(a,\varphi,\Omega) \tag{3.28}$$

$$\langle\dot{\varphi}\rangle = \frac{n}{am\Omega}\left[\frac{1}{2}\left(\omega_0^2 - \frac{m^2}{n^2}\Omega^2\right)a + \varepsilon Q(a,\varphi,\Omega)\right] \tag{3.29}$$

where it is denoted

$$S(a,\varphi,\Omega) = -\left\langle f^{tb}\sin\phi\right\rangle \tag{3.30}$$

$$Q(a,\varphi,\Omega) = -\left\langle f^{tb}\cos\phi \right\rangle \tag{3.31}$$

Here $<\cdot>$ is the averaging operator symbol over period *T*. The stationary solutions a and φ of Eqs. (3.28), (3.29), are then defined by the condition

$$\langle \dot{a} \rangle = 0 \rightarrow S(a,\varphi,\Omega) = 0 \tag{3.32}$$

$$\langle \dot{\varphi} \rangle = 0 \rightarrow \frac{1}{2}\left(\omega_0^2 - \frac{m^2}{n^2}\Omega^2\right)a + \varepsilon Q(a,\varphi,\Omega) = 0 \tag{3.33}$$

3.3. Application of averaging method to a mono-stable Duffing piezoelectric energy harvester system subjected to base excitation harmonic with a single degree of freedom lumped parameter model in some nonlinear resonance

3.3.1. Primary resonance of piezoelectric Duffing energy harvester

Two first-order approximate solutions of amplitude-frequency curve corresponding to the minus and plus signs, respectively:

$$\Omega^2 = \omega_0^2 + \varepsilon\left(\frac{3}{4}\gamma a^2 + \frac{\kappa^2\Omega^2}{\alpha^2+\Omega^2}\right) \pm \varepsilon\sqrt{\frac{A^2\Omega^4}{a^2} - \left(2\xi\Omega + \frac{\kappa^2\alpha\Omega}{\alpha^2+\Omega^2}\right)^2} \tag{3.56}$$

The useful output electrical power delivered to the resistive load is:

$$P_{use-main} = \varepsilon\kappa^2 M\alpha a^2\left[\frac{\Omega^2}{\alpha^2+\Omega^2}\cos(\Omega t+\varphi) - \frac{\alpha\Omega}{\alpha^2+\Omega^2}\sin(\Omega t+\varphi)\right]^2 \tag{3.69}$$

The useful output electrical energy delivered to the resistive load

$$E_{use-Main} = \int_0^{\pi/\Omega} P_{use}dt = \frac{\pi\varepsilon M\Omega}{2\left(\Omega^2+\alpha^2\right)}\alpha\kappa^2 a^2 \tag{3.70}$$

the input mechanical power is defined as:

$$P_{in-main} = \\ = M\left\{\begin{array}{l} \varepsilon\left[\begin{array}{l} 2\xi a\Omega\sin(\Omega t+\varphi) - \gamma a^3\cos^3(\Omega t+\varphi) \\ -\dfrac{\kappa^2 a\Omega^2}{\alpha^2+\Omega^2}\cos(\Omega t+\varphi) + \\ +\dfrac{\alpha\kappa^2 a\Omega}{\alpha^2+\Omega^2}\sin(\Omega t+\varphi) \end{array}\right] \\ -\omega_0^2 a\cos(\Omega t+\varphi) \end{array}\right\} \times \left(-\varepsilon\Omega A\cos\Omega t\right) \tag{3.73}$$

the input mechanical energy of Duffing PEH system at the excitation frequency Ω_S with the peak amplitude a_{peak}

$$E_{in-Main} = \pi\varepsilon M\Omega_s a_{peak}^2\left(\xi+\kappa^2\frac{\alpha}{2\left(\alpha^2+\Omega_S^2\right)}\right) \tag{3.74}$$

The efficiency of the mono-stable Duffing PEH system at the excitation frequency Ω_S with the peak amplitude a_{peak}

$$\eta_{Main}^{peak} = \frac{\alpha\kappa^2}{\left[2\xi\left({\Omega_S}^2+\alpha^2\right)+\alpha\kappa^2\right]} \tag{3.75}$$

3.3.2. Sub-harmonic resonance of piezoelectric Duffing energy harvester

the case of sub-harmonic resonance: $(m;n)=(1;3)$ (3.79)

the stationary solution (a,φ) is determined from the system of equations:

$$\begin{aligned}&\left\{\left(\omega_0^2-\frac{\Omega^2}{9}\right)+\frac{3}{4}\varepsilon\gamma\left(a^2+2f_0^2\right)+\frac{\varepsilon\Omega^2\kappa^2}{\Omega^2+9\alpha^2}\right\}^2+\\&+\left[\frac{2}{3}\varepsilon\xi\Omega+\frac{3\varepsilon\alpha\kappa^2\Omega}{\Omega^2+9\alpha^2}\right]^2-\frac{9}{16}\varepsilon^2\gamma^2a^2f_0^2=0\end{aligned} \tag{3.86}$$

the relation of amplitude-frequency:

$$\begin{aligned}a^2 &= \frac{4}{27\varepsilon\gamma}\left(\Omega^2-9\omega_0^2\right)-\frac{3}{2}f_0^2-\frac{4\Omega^2\kappa^2}{3\gamma\left(\Omega^2+9\alpha^2\right)}\pm\\&\pm\frac{2}{9}\left\{\begin{aligned}&\frac{4}{(3\varepsilon\gamma)^2}\left[9\omega_0^2-\Omega^2+\frac{9\varepsilon\Omega^2\kappa^2}{\Omega^2+9\alpha^2}+\frac{81\varepsilon\gamma}{8}f_0^2\right]^2-\frac{3\varepsilon}{\gamma}\left(2\xi\Omega+\frac{9\alpha\kappa^2\Omega}{\Omega^2+9\alpha^2}\right)^2\\&-\frac{1}{3\varepsilon\gamma}\left(9\omega_0^2-\Omega^2+\frac{9\varepsilon\Omega^2\kappa^2}{\Omega^2+9\alpha^2}+\frac{27\varepsilon\gamma f_0^2}{2}\right)^2\end{aligned}\right\}^{1/2}\end{aligned} \tag{3.87}$$

The voltage response in sub-harmonic resonance of PEHs one has

$$v(t) = -\frac{3a\Omega}{9\alpha^2+\Omega^2}\left[\alpha\sin\left(\frac{\Omega}{3}t+\varphi\right)-\frac{1}{3}\Omega\cos\left(\frac{\Omega}{3}t+\varphi\right)\right] + \frac{\varepsilon A\Omega^3}{\left(\omega_0^2-\Omega^2\right)\left(\alpha^2+\Omega^2\right)}\left(\Omega\cos\Omega t-\alpha\sin\Omega t\right) \tag{3.89}$$

The power consumed by the external resistor is calculated as

$$P_{use-Sub} = \varepsilon\frac{V^2}{R} = \varepsilon\frac{\theta^2}{R{C_p}^2}v^2(t) = \varepsilon\alpha\kappa^2 M v^2(t) \tag{3.90}$$

the PEHs to the external load or the electrical energy generated per cycle

$$E_{use-Sub} = \frac{3\pi}{2}\varepsilon\alpha\kappa^2\Omega M\left[\frac{\varepsilon^2A^2\Omega^4}{\left(\Omega^2+\alpha^2\right)\left(\Omega^2-\omega_0^2\right)^2}+\frac{a^2}{\left(\Omega^2+9\alpha^2\right)}\right] \tag{3.91}$$

The input mechanical energy one has as (3.92)

Work done by the external excitation on the system per one cycle can be derived as the input mechanical energy over the period

$$W_{in-Sub} = \frac{3\pi\varepsilon^2AM\Omega}{2}\left\{-2\xi f_0-\frac{1}{f_0}\left[\frac{\alpha\kappa^2a^2}{\left(\Omega^2+9\alpha^2\right)}+\frac{2\xi a^2}{9}\right]-\frac{\alpha\kappa^2 f_0}{\left(\Omega^2+\alpha^2\right)}\right\} \tag{3.94}$$

Efficiency:
$$\eta_{Sub} = \frac{E_{use-Sub}}{E_{in-Sub}} \tag{3.98}$$

3.3.3. Super-harmonic resonance of piezoelectric Duffing energy harvester

In the super-harmonic resonance analysis of Duffing system, we use the first approximation between ω_0 and Ω:

$$\omega_0^2 = \frac{m^2}{n^2}\Omega^2-\varepsilon\sigma,\ (m,n)=(3,1);\sigma=\frac{1}{\varepsilon}\left(\omega_0^2-\frac{m^2}{n^2}\Omega^2\right) \tag{3.100}$$

The following amplitude-frequency equation:

$$9\Omega^2a^2\left(2\xi+\frac{\alpha\kappa^2}{9\Omega^2+\alpha^2}\right)^2 + a^2\left(\sigma-\frac{3\gamma}{4}a^2-\frac{3}{2}{f_0}^2\gamma-\frac{9\Omega^2\kappa^2}{9\Omega^2+\alpha^2}\right)^2-\frac{\gamma^2{f_0}^6}{16}=0 \tag{3.108}$$

The amplitude-frequency equation (3.108) reduces to the one for Duffing mechanical system when the electromechanical coupling coefficient $\kappa^2 = 0; \alpha = 0$.The relation of amplitude-frequency corresponding to two first-order approximate solutions to the minus and plus signs, respectively

$$\Omega^2_{(1)I} = \frac{\omega_0^2}{9} + \frac{\varepsilon}{9}\left(\frac{3\gamma}{4} a^2 + \frac{3\varepsilon^2 \gamma}{128} A^2 + \frac{\omega_0^2 \kappa^2}{\omega_0^2 + \alpha^2} \right) \mp \frac{\varepsilon}{9} \sqrt{\frac{\varepsilon^6 A^6 \gamma^2}{2^{22} a^2} - \omega_0^2 \left(2\xi + \frac{\alpha \kappa^2}{\omega_0^2 + \alpha^2} \right)^2} \quad (3.109), (3.110)$$

The useful output electrical power delivered to the resistive load is

$$P_{use-\sup er} = \varepsilon \alpha \kappa^2 M \Omega \left[\frac{3a}{9\Omega^2 + \alpha^2} \left(3\Omega \cos\left(3\Omega t + \varphi\right) - \alpha \sin\left(3\Omega t + \varphi\right) \right) + \frac{\varepsilon A \Omega^3}{(\Omega^2 + \alpha^2)(\omega_0^2 - \Omega^2)} \left(\Omega \cos \Omega t - \alpha \sin \Omega t \right) \right]^2 \quad (3.120)$$

the useful output electrical energy delivered to the resistive load:

$$E_{use-Super} = \frac{\pi \varepsilon M \Omega}{2} \alpha \kappa^2 \left\{ \frac{9a^2}{\left(9\Omega^2 + \alpha^2\right)} + \frac{\varepsilon^2 A^2 \Omega^4}{(\Omega^2 + \alpha^2)(\Omega^2 - \omega_0^2)^2} \right\} \quad (3.121)$$

The input mechanical power is defined as eq.(3.124), and
the input mechanical energy over the period $T = \pi / \Omega$:

$$\mathrm{W}_{in-Super} = \frac{\pi \varepsilon^2 MA}{2} \Omega f_0 \left[-2\xi \left(1 + \frac{9a^2}{f_0^2} \right) - \alpha \kappa^2 \left(\frac{9a^2}{\left(9\Omega^2 + \alpha^2\right) f_0^2} + \frac{1}{\left(\Omega^2 + \alpha^2\right)} \right) \right] \quad (3.125)$$

The efficiency of the mono-stable Duffing PEH system at super-harmonic resonance are presented as follows

$$\eta_{Super} = \frac{\alpha \kappa^2 \left\{ \frac{9a^2}{\left(9\Omega^2 + \alpha^2\right)} + \frac{f_0{}^2}{(\Omega^2 + \alpha^2)} \right\}}{\varepsilon A f_0 \left[2\xi \left(1 + \frac{9a^2}{f_0{}^2} \right) + \alpha \kappa^2 \left(\frac{9a^2}{\left(9\Omega^2 + \alpha^2\right) f_0{}^2} - \frac{1}{\left(\Omega^2 + \alpha^2\right)} \right) \right]} \quad (3.127)$$

In Chapter 3, the thesis presented the following main results:

1. The thesis has developed and extended the averaging method used for a nonlinear electromechanical system subjected to base excitation with a Single degree of freedom lumped parameter model.
2. From the content of the development and expansion of the average method used for nonlinear electromechanical systems, the thesis has determined the visible expressions of the amplitude-frequency relationship, displacement, voltage electrical responses, input-output mechanical power, input-output potential useful electrical energy, and the efficiency of the mono-stable Duffing PEHs at primary resonance, secondary resonance, and linear system corresponding for comparison.
3. The thesis has verified the content of developing the average method used for nonlinear electromechanical systems.

Chapter 4. THE ANALYSIS OF THE INFLUENNCE OF PARAMETER ON THE MONO-SATBLE DUFFING PEH SYTEM AT NONLINEAR RESONONCE EFFECTS

4.1. *Test and Survey Results*

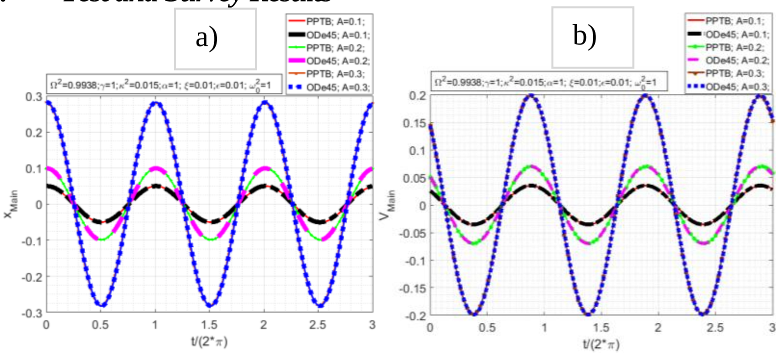

Fig 4. 1. Correlation between displacement, voltage response using a solution numerical simulation and averaging method of the mono-stable Duffing PEH system at super-harmonic resonance over time with $a_{\ddot{z}_{peak}} = \varepsilon A\Omega^2$

Investigate when changing the excitation amplitude with the same excitation frequency in the vicinity of the main harmonic resonance ($\Omega^2 = 0.9938$) and keeping the other parameter values of the system unchanged, in order to evaluate the influence of the excitation amplitude affect displacement and voltage response. As shown in Fig 4.1 a) and Fig 4.1

b), it is clear that the curves representing numerical results are almost identical (asymptotic) to those obtained from the averaging method. At the same time, numerical and analytical results have the same conclusion that: displacement response amplitude and voltage increase sharply when increasing the amplitude of background agitation. When changing the excitation amplitude, the maximum error of displacement response amplitude and maximum voltage between the numerical method and the average method is 0.035% and 0.379%, respectively, as detailed in Table 4.1.

Table 4. 1. compare between numerical method and average method when changing base excitation amplitude

Parameter	Ode45	Average method	Err (%)	Ode45	Average method	Err (%)	Ode45	Average method	Err (%)
	A=0.1			A=0.2			A=0.3		
$\max\lvert\bar{x}_1\rvert$	0.050	0.050	0.028	0.099	0.099	0.0402	0.282	0.282	**0.035**
$\max\lvert\bar{x}_3\rvert$	0.035	0.0353	0.003	0.069	0.069	0.007	0.199	0.199	**0.379**

The numerical calculation results are consistent with the mechanical laws, this has the effect of further confirming the suitability of the algorithm and the reliability of the calculated program.

4.2. Analysis and investigation of the influence of parameters on the mono-stable Duffing PEH system subjected to harmonic base excitation with a single-degree-of-freedom lumped mass model in the main harmonic resonance

The corresponding values of parameters as

$$\begin{aligned}&\varepsilon = 0.01; \omega_0 = 1; \xi = (0.01; 0.1; 0.3); \gamma = (0.15; 0.3; 0.5; 1);\\&\alpha = (0.05; 0.5; 1); \kappa^2 = 0.015; A = (0.1; 0.3; 0.5);\end{aligned} \tag{4.3}$$

The branches represent the amplitude-frequency curve of the electromechanical system under investigation at the effect main resonance is similar to the superharmonic resonance and the mechanical system. That confirms the electromechanical parameters (the normalized resistance coefficient, electromechanical coupling coefficient) do not affect the amplitude-frequency curve form.(*Fig. 4.5 và Fig. 4.7*). Obviously, the frequency range of the nonlinear electromechanical system at the main resonant has larger than the corresponding linear, allowing for the expansion of the working frequency range of the piezoelectric energy harvester. As shown in *Fig 4.7*, extreme coordinates $(a_{peak};\Omega_S)$ have a decrease in the jump when increasing the value of the electromechanical coupling coefficient κ^2, and the line branches representing the amplitude-frequency relationship tend to be strongly inclined to the right of the graph when the electromechanical coupling coefficient is decreased.

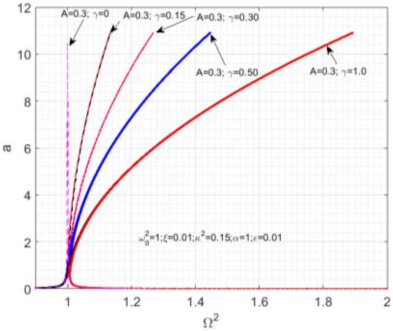

Fig 4. 5. Amplitude–frequency relation of the mono-stable Duffing PEH system at main-harmonic resonance with *γ=0.0; γ=0.15; γ=0.3; γ=0.5; γ=1;*

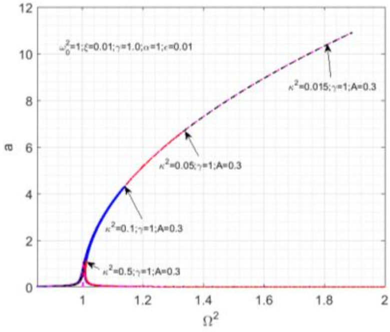

Fig 4. 7. Amplitude–frequency relation of the mono-stable Duffing PEH system at main-harmonic resonance with κ^2

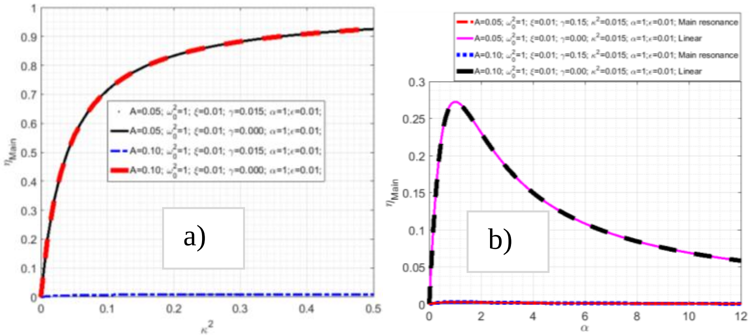

Fig.4.16. Efficiency of the mono-stable Duffing PEH system at main harmonic and linear system corresponding with κ^2 and α

As shown in *Fig 4. 16,* the efficiency of the linear electromechanical system is much larger than that of the corresponding nonlinear electromechanical system when the same set of input survey parameters, besides the efficiency is a linear increasing function for with nonlinear electromechanical systems. Specifically, with the same value of background excitation amplitude, the performance curves of the linear system are an increasing function of the electromechanical bonding coefficient, increasing sharply when the electromechanical bonding coefficient is in the range and then slightly increasing with the increase of the electromechanical bonding coefficient (*Fig 4. 16.* a)). Investigating the effect of piezoelectric coefficient, the curve is a strong increasing function, peaking when then decreasing linearly (*Fig 4.16* b)).

4.3. Analysis and investigation of the influence of parameters on the mono-stable Duffing PEH system subjected to harmonic base excitation with a single-degree-of-freedom lumped mass model in the sub-harmonic resonance

The corresponding values of parameters as

$$\varepsilon = 0.01; \omega_0 = 1; \xi = (0.01; 0.02; 0.3); \gamma = (0.05; 0.5; 1);$$
$$\alpha = (0.05; 0.5; 1); \kappa^2 = 0.015; A = (10; 20; 30; 50); \tag{4.4}$$

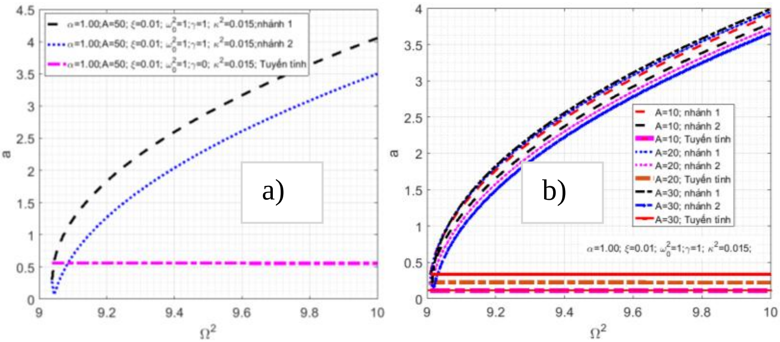

Fig 4. 24. Amplitude–frequency relation of the mono-stable Duffing PEH system at sub-harmonic resonance and linear system corresponding

The results of the survey on the amplitude-frequency curve relationship of the electromechanical system show that, with the same value of the background excitation amplitude, the two branches of the amplitude-frequency curve of the nonlinear electromechanical system are above the linear system. When the amplitude of the base increases (*Fig 4.24*.b) and (*Fig 4.24*.a) the linear system line intersects the branches of the mono-stable Duffing PEH system at sub-harmonic resonance at two points (corresponding to the two branches of the line nonlinear electromechanical system).

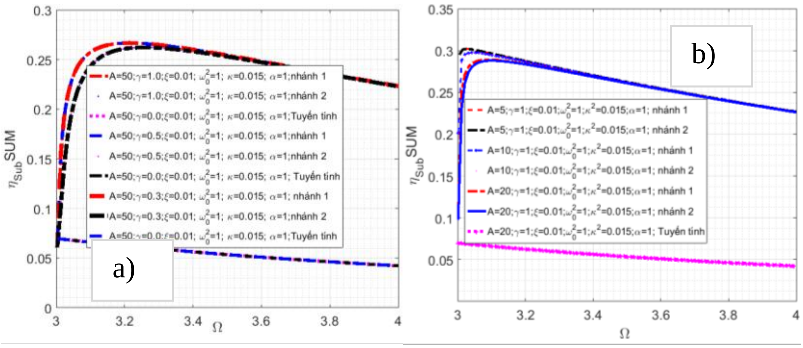

Fig 4. 32. Efficiency of the mono-stable Duffing PEH system at sub-harmonic and linear system corresponding

The two branches curve the efficiency of the mono-stable Duffing PEH system at sub-harmonic correspond to two values of amplitude-frequency and linearity respectively, presented in *Fig 4.32*. a) and *Fig 4.32*. b). The efficiency curve of the nonlinear system has a peak, which peaks when the excitation frequency is near the resonant frequency, and after the peak, the nonlinear system efficiency decreases linearly similar to the curve of a linear system. however, is still much larger than the corresponding linear system in both cases with different values of cubic nonlinear coefficients and excitation amplitudes. The efficiency of the mono-stable Duffing PEH system at sub-harmonic slightly decreases as the value of base excitation amplitude increases.

4.4. Analysis and investigation of the influence of parameters on the mono-stable Duffing PEH system subjected to harmonic base excitation with a single-degree-of-freedom lumped mass model in the super-harmonic resonance

Parameters the thesis selects the survey

$$\begin{aligned} &\varepsilon = 0.01; \omega_0 = 1; \xi = (0.01; 0.1; 0.3); \gamma = (0.3; 0.5; 1); \\ &\alpha = (0.05; 0.5; 1); \kappa^2 = 0.015; A = (300; 500; 700); \end{aligned} \tag{4.2}$$

As shown in *Fig4. 39.* and *Fig 4. 40.*, the peak amplitude value of the displacement response of the mono-stable Duffing PEH system at the super-harmonic resonance effect is proportional to the price increase values of base-excitation amplitude and cubic nonlinear coefficient, and at the same time much larger than the corresponding linear system amplitude response.

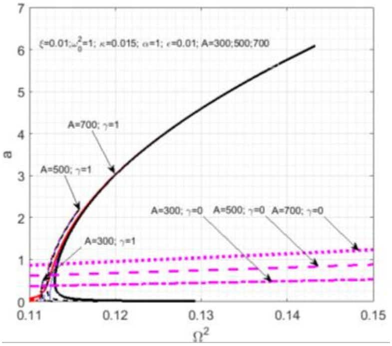

Fig 4. 39. Amplitude–frequency relation of the mono-stable Duffing PEH system at super-harmonic and linear system corresponding

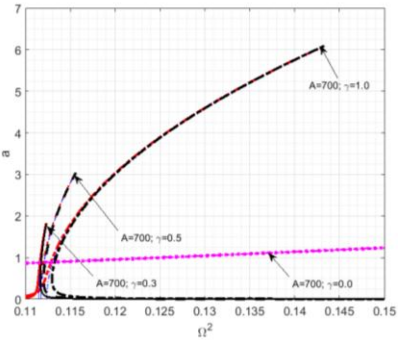

Fig 4. 40. Amplitude–frequency relation of the mono-stable Duffing PEH system super-harmonic and linear system corresponding with $\gamma=0.3$; $\gamma=0.5$; $\gamma=1$

When the value of the base excitation amplitude and the cubic nonlinear coefficient increases, the curves representing the amplitude-frequency relationship of the nonlinear electromechanical system tend to lean to the right of the graph, obviously, in this case, the working frequency range of the nonlinear electromechanical system is extended in the superharmonic resonance. In the cases where the value of background excitation amplitude and cubic nonlinear coefficient increases the extreme point on the frequency axis Ω^2 inclined to the right, the coordinate graph $Oa\Omega^2$ follows the inclined trend of the curve branches.

The efficiency of the mono-stable Duffing PEH system at super-harmonic tends to decrease with the minimum at the peak amplitude, the coordinates of the minimum point of the efficiency tend to shift towards the left side of the graph, with the jump increasing markedly as the cubic nonlinear coefficient decreases, and at the same time the frequency band near the superharmonic resonance region of the system is significantly narrowed (*Fig 4.50*.a) and *Fig 4.50*. b)). The effect of the base excitation amplitude on efficient energy harvesting is similar to the cubic nonlinear coefficient, but the minimum point of the efficiency does not have a large jump and shifts significantly to the left of the graph.

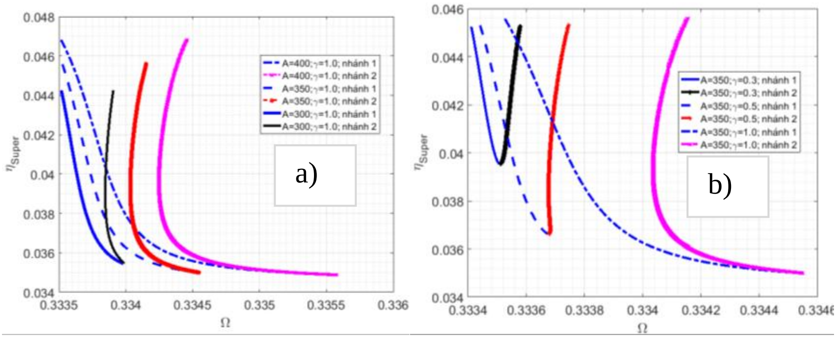

Fig. 4.50 Efficiency of the mono-stable Duffing PEH system at super-harmonic and linear system corresponding

Conclusion Chapter 4

The survey results show the frequency range near the vicinity of the area approach the resonance of the nonlinear electromechanical system wider than the corresponding linear system; The efficiency of the mono-stable Duffing PEH system in the research thesis is larger in the vicinity of the sub-harmonic resonance but smaller in the vicinity of the superharmonic and main resonance compared with the corresponding linear system. However, the input mechanical power, output, input mechanical energy, and potentially useful electrical energy of the electromechanical system the thesis studied in the resonance effect is larger than the linear system;

CONCLUDING REMARKS

1) The thesis has established a system of nonlinear differential equations for a set of energy harvesters in the form of Duffing-type nonlinear oscillations of cantilever beams with two piezoelectric layers (an upper and a lower layer covering the entire surface of the substructure - the bi-morph piezoelectric cantilever) under harmonic excitation;
2) The thesis also includes modeling a set of devices, development, and verification of the reliability of applying the averaging method to a nonlinear piezoelectric energy harvester system. The objective is to establish and determine explicit analytical expressions for the relationship between amplitude and frequency, displacement response, voltage, and energy harvesting efficiency in the resonance domain including primary, secondary resonances, and linear respectively. This is achieved by utilizing the lumped mass model as a single-degree-of-freedom system under harmonic excitation;
3) Furthermore, the thesis has analyzed, surveyed, and evaluated the influence of parameters on displacement responses, voltage, input mechanical power, output, input mechanical energy, output potential useful electrical energy, and efficiency of energy harvesting in nonlinear resonance effects, thus drawing useful evaluation conclusions;

LIST OF THE PUBLICATIONS RELATED TO THE DISSERTATION

[1] Anh, N. D., Linh, N. N., **Van Manh, N**., Tuan, V. A., Van Kuu, N., Nguyen, A. T., & Elishakoff, I. (2020). *Efficiency of mono-stable piezoelectric Duffing energy harvester in the secondary resonances by averaging method. Part 1: Sub-harmonic resonance.* International Journal of Non-Linear Mechanics, Volume 126, November 2020, 103537, https://doi.org/10.1016/j.ijnonlinmec.2020.103537. (ISI)

[2] Linh, N. N., Nguyen, A. T., **Van Manh, N**., Tuan, V. A., Van Kuu, N., Anh, N. D., & Elishakoff, I. (2021). *Efficiency of mono-stable piezoelectric Duffing energy harvester in the secondary resonances by averaging method, Part 2: Super-harmonic resonance*. International Journal of Non-Linear Mechanics, Volume 137, December 2021, 103817, 31/08/2022, https://doi.org/10.1016/j.ijnonlinmec.2021.103817. (ISI)

[3] Nguyen Ngoc Linh, **Nguyen Van Manh**, Vu Anh Tuan, *Analysis of main resonace for a nonlinear piezoelectric energy harvester by averaging method*, Tạp chí cơ khí Việt Nam, số đặc biệt, 10/2020, Trang 434-439.

[4] Nguyen Ngoc Linh, Nguyen Dong Anh, **Nguyen Van Manh**, *Mô hình khối lượng tập trung của bộ thu thập năng lượng có kết cấu dầm công xôn với hai lớp áp điện phi tuyến.* Tuyển tập công trình khoa học Hội nghị Cơ học toàn quốc lần thứ XI, Hà Nội, 2-3/12/2022, tập II. ISBN 978-604-357-085-4, trang 127-136.

[5] **Nguyen Van Manh**, Nguyen Dong Anh, Nguyen Ngoc Linh, *Hiệu suất của bộ thu thập năng lượng áp điện lên hệ phi tuyến kiểu Duffing – Trường hợp cộng hưởng chính.* Tuyển tập công trình khoa học Hội nghị Cơ học toàn quốc lần thứ XI, Hà Nội, 2-3/12/2022, tập II. ISBN 978-604-357-085-4, trang 73-83.