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**THE STUDY PROPOSES A COMPLEX FUZZY TRANSFER
LEARNING SYSTEM BASED ON SUBSPACE SAMPLING
TECHNIQUE AND DIRECTED GRAPH STRUCTURE**

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LIST OF THE PUBLICATIONS RELATED TO THE DISSERTATION

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PREFACE

1. The significance of the dissertation

Inference is reaching new conclusions or making judgments based on given information. It is vital and commonly used in everyday life and various fields. In everyday life, inference assists people in making judgments based on available knowledge, identifying options, and solving complicated problems. It can help anticipate market trends and make production and marketing decisions in business. In the sphere of science and research, deduction aids in the development and testing of theories, as well as the drawing of conclusions based on existing evidence and information. In artificial intelligence (AI), computers utilize inference to interact with data, learning from the data to help generate predictions or choices.

To solve the problem of uncertainty and ambiguity and the periodicity and frequency present in the data, Ramot et al. [1] added the phase element and proposed the concept of complex fuzzy sets (CFS). Many studies have suggested extensions of complex fuzzy inference systems, such as the ANCFIS model (Adaptive Neural Fuzzy Inference System) [2] and the extended versions (ANCFIS-ELM and FANCFIS [3, 4]), which are combinations of complex fuzzy systems and neural networks. The Mamdani complex fuzzy inference system (M-CFIS) [5] with an inference structure based on CFS and decision support system applications was introduced recently. Based on the M-CFIS model, two improvements, including rule reduction for the M-CFIS system [6] and M-CFIS for the knowledge graph [7], have been proposed to improve the training and testing process on M-CFIS. The investigations above are the most common CFIS that handles cyclical data in knowledge systems.

Transfer learning (TL) is a strategy whereby performance on a related task is improved by applying knowledge gained from other tasks to achieve two goals.:

- Use existing knowledge to solve the shortening problems of knowledge of new learning tasks.
- Reduce the time that you spend learning new tasks.

In circumstances where data and expertise are few, transfer learning approaches used in machine learning and artificial intelligence have resulted in improved performance [8–11]. TL has achieved impressive results in text classification [12], image processing [13, 14], medical diagnosis [15–19], object detection and behavior recognition [20–24], network model [25] and stock market forecast [26] etc.

Although TL is rapidly evolving, unclear and ambiguous information in issues remains a fundamental constraint in training and creating models [26]. To solve these problems, Jethro and Simon [27] introduced fuzzy transfer learning (FTL), which combined TL with fuzzy theory to aim at transferring uncertain, ambiguous information. The fuzzy set (FS) theory is a popular and effective tool for handling vague and uncertain data. Combining fuzzy theory with transfer learning has solved situations of information deficiency, accompanied by ambiguity and uncertainty [26, 28, 29].

Despite breakthroughs in complex fuzzy inference systems and fuzzy transfer learning, several restrictions remain problems, including:

- The M-CFIS model is typical for handling data with uncertain and periodic elements. However, the main drawback of this model is that the rule base is

created directly from the entire data without actually learning. Such models often have poor adaptability and difficulty processing complex information, where relationships between data are unclear or change over time.

- In addition, the M-CFIS system's ability to handle data with periodic and frequency uncertainties limits processing time. The phase component generated during the description of cyclic/periodic factors increases processing time.
- In cases where data is increasing and being updated continuously, building an M-CFIS model according to the traditional law update plan is not feasible and takes too much time.
- Most current FTL systems stop at combining TL techniques with traditional fuzzy logic. In recent studies, there has been little or even no research on FTL on extended, incredibly complex fuzzy sets.

Complex fuzzy theory and inference are meaningful in solving problems that are ambiguous, uncertain, and cyclic or periodic. However, the scope of the application is limited due to the time factor. Meanwhile, transfer learning techniques can reduce learning time. Therefore, this thesis specifies the research task of solving the limitations of CFIS in terms of time (completing research on complex fuzzy inference systems) based on transfer learning techniques. Specifically, they improve the time efficiency for building complex fuzzy inference models for the target domain (the data domain in which the complex fuzzy inference system needs to be made) and the inference time of the inference system.

2. Research objectives of the thesis

2.1. The general objective of the thesis

The general goal of the thesis is to research and develop a transfer learning system based on the complex fuzzy inference model to take advantage of existing knowledge in previous models and minimize the time to build a complex fuzzy inference system in the target domain.

2.2. Specific Objectives

Starting from the general goal, the specific goals of the thesis include:

- Propose a transfer learning model based on the Mamdani complex fuzzy inference system (M-CFIS), applying it to build a complex fuzzy inference system for the target domain.
- Proposing a way to represent knowledge (complex fuzzy rules) on a new data structure to improve inference operations in time and support quick and effective knowledge transfer.

3. Object and Scope of the Thesis

3.1. The object of Study

The research object of the thesis is inference systems following the complex fuzzy set approach and transfer learning techniques.

3.2. Scope of Study

Based on the research objectives and content, the research scope of the thesis is determined as follows:

- *Theoretically*: Research on transfer learning models on complex fuzzy inference systems in the context that the source and target tasks are the same, the source and target tasks have the same distribution, the same number of attributes but different value ranges of the attributes.
- *Experiment*: Transfer learning experiments on complex fuzzy inference systems are used in cases where the source task and target task are the same, the source task and target task have the same distribution, and the same number of attributes but different ranges of values of the attributes' properties.

4. Methodology and Research Content

4.1. Research Methodology

The research method of the thesis is theoretical research and experimental research

- Theoretical research: Overview of complex fuzzy set theory, complex fuzzy inference systems, transfer learning techniques and models, analysis of advantages, disadvantages, and remaining problems of related research. Summary of related research on fuzzy sets, complex fuzzy sets, fuzzy inference systems, complex fuzzy inference systems and transfer learning techniques, and fuzzy transfer learning. We propose a transfer learning model based on a complex fuzzy inference system and improved methods to achieve the goals.
- Experimental research: The proposed models and algorithms are installed, tested, compared, and evaluated with the corresponding model using the traditional method on sample data sets from the UCI data warehouse and real data sets at Thai Nguyen Iron and Steel Hospital to demonstrate the effectiveness of theoretical research.

4.2. Research Content

To achieve the research goals, the thesis focuses on the following main contents:

- Research on developing a complex fuzzy inference system based on transfer learning techniques
- Research on the graph structure representing complex fuzzy rules for merging and inference in the rule adaptation process on the complex fuzzy transfer learning system.

5. Contribution of the Thesis

- **Proposed novel transfer learning model on complex fuzzy inference system**
 - The proposed transfer learning model reuses knowledge obtained from CFIS of a related domain (called the source domain). Combining transfer learning techniques and the CFIS system's inference mechanism reduces the time to build a CFIS system for another domain (called the target domain).
 - The proposed model includes the stages 1) data selection from the target domain, 2) input domain correction, 3) rule adaptation, and 4) rule synthesis.

- Theoretical and experimental results show the ability to improve time when building a complex fuzzy inference system for the target domain using transfer learning techniques and complex fuzzy inference system mechanisms. This contributes significantly to expanding the application scope of the complex fuzzy inference model in cases of limited time constraints or cases of extensive and continuously updated data.
- These contributions are presented in the content **Chapter2** of the thesis.
- **Proposing a new data structure - CFRG representing a set of complex fuzzy rules applied to transfer learning models on complex fuzzy inference systems**
 - The CFRG structure is proposed to represent a set of complex fuzzy rules for the task of complex fuzzy inference, helping to reduce the time of the complex fuzzy inference process. Increase the applicability of complex fuzzy inference models for real-world problems, especially with extensive and continuously updated data.
 - The CFRG structure represents the amplitude and phase components of the rule on each node, making it easy to select values when editing the rule during the rule adaptation process, speeding up the adaptation time and increasing the accuracy of the model.
 - These contributions are presented in detail in **Chapter 3** of the thesis.

6. The layout of the thesis

Thesis “**Research proposing a complex fuzzy transfer learning system based on subspace sampling technique and directed graph structure**” includes an introduction, three content chapters, a conclusion and a list of references with the following main contents:

- **Introduction:** Introduce the context of the research and evaluate the role and capabilities of inference systems, complex fuzzy inference systems, transfer learning techniques as well as their limitations; research issues; Objectives of the study; research approaches and methods; research content; scope and limitations of the study; Main contributions and structure of the thesis.
- **Chapter 1:** Presents basic knowledge for the research thesis, including Concepts of fuzzy sets, complex fuzzy sets, complex fuzzy inference systems, transfer learning models, and fuzzy transfer learning. The research problem, data, and experimental environment are also introduced in this chapter.
- **Chapter 2:** Presents in detail the process of building a transfer learning model on a complex fuzzy inference system (including four stages) and experimental results on UCI data sets and real data sets along with the Analyze and evaluate the proposed model.
- **Chapter 3:** Presenting the proposed CFRG structure applied to complex fuzzy inference and rule adaptation in the transfer learning model proposed in Chapter 2, experimental results, analysis, and performance evaluation.
- **Conclusion and development direction:** Presents the thesis results, limitations and development directions.

Chapter 1

RESEARCH OVERVIEW AND THEORETICAL BASIS

In this first chapter, the thesis presents the general theory of inference systems, fuzzy inference systems, and complex fuzzy inference systems — the research object of the thesis. Next, the thesis presents the transfer learning technique used in the research process to address the limitations of complex fuzzy inference systems. At the same time, this chapter also introduces an overview of the data sets, environments, experimental scenarios, and experimental evaluation metrics.

1.1 Inference and complex fuzzy inference systems

Deduction is considered one of the critical tools and techniques of science and life. Deduction is drawing new information and conclusions from known information or logical principles. We can connect facts, rules, and information through inference to create a more comprehensive and detailed view of things and phenomena. Deduction is also the basis for making logical and reasoned arguments. Especially in artificial intelligence and machine learning, inference is central. Computers and intelligent systems are programmed to reason from data to perform forecasts and trend analysis, understand and automate decisions, support decision-making, etc.

Inference systems play an essential role in intelligent systems and the fields of machine learning and artificial intelligence (AI) because of their ability to process uncertain information and make decisions based on inference rules. Inference systems are widely used in recommendation systems to recommend products, services, or content based on user history and behavior [30, 31]. Inference systems can handle fuzzy and ambiguous information, helping improve the user experience and increase interactivity. Not only that, inference systems are an essential component in decision support systems, helping decision makers process complex information and make decisions based on predefined rules [5, 32, 33].

The effectiveness of inference is based on historical information and data. Meanwhile, information and data are constantly changing and increasingly contain new characteristics. New characteristics in data, such as ambiguity and frequency/period, have made traditional logic more accessible and more effective in the inference process. Research based on FIS [34] can solve situations where information and data are ambiguous and uncertain. A CFIS [1, 35] deals with the data with periodicity and seasonal. Different inference methods are used depending on the other problems. Complex fuzzy inference systems [35], also known as complex fuzzy logic systems (CFLS), are based on FISs and are known for their ability to handle linguistic concepts related to knowledge of frequency and periodicity factors. That is, the CFIS is suitable for problems related to data that is uncertain, ambiguous, and has frequency and periodicity [5, 7].

1.2 Review of related literature and studies

1.2.1 Fuzzy inference models

Fuzzy logic has always been mentioned as a tool to describe uncertain and ambiguous information. It is commonly applied to solving problems related to

prediction, control, pattern detection, and decision support systems with uncertain information. It is also considered a computational model that is capable of simultaneously processing both linguistic knowledge and numerical data. Fuzzy logic helps computers understand and imitate human thinking, with the goal of increasing the efficiency of the decision-making process with ambiguous, uncertain knowledge. The fuzzy logic theory has created a series of FISs [2, 3, 5, 36, 37]. Each FIS is described as a nonlinear mapping to produce results based on ambiguous arguments and a set of IF-THEN fuzzy rules.

1.2.2 Transfer learning and fuzzy transfer learning

TL in machine learning is inspired by human transfer learning ability when taking advantage of existing knowledge of related domains (called source domains) to improve learning performance or reduce the number of labeled samples that must be present in the target domain [11, 38].

Taxonomy of transfer learning

Research on TL is based on different bases, such as problems, data labels, and models. However, every transfer learning process considers the problems being transferred. Three questions are always asked in transfer learning:

1. What is transferred?
2. How to transfer?
3. When will it be transferred?

According to [11] answers to the questions "Transfer of what?" and "How to transfer?" have created a number of different TL research directions, such as classifying transfer learning according to the level of labeling by domain and by transfer solution. In addition to the above classification of transfer learning, there are other ways of classifying transfer learning, such as transfer learning based on model strategy, transfer learning using deep learning technology, fuzzy transfer learning, etc.

Fuzzy transfer learning

Data is the basis of most machine-learning techniques, whereas real-world information and data are often uncertain and ambiguous. When information and data are uncertain or vague, learning methods will include different techniques to represent it and obtain the necessary knowledge for the learning stage. FTL is a combination of fuzzy theory and transfer learning techniques to solve problems of data scarcity and uncertain and ambiguous data. Meanwhile, TL techniques will solve the problem of knowledge deficiency in the target domain by taking advantage of knowledge in related source domains. Fuzzy theory helps describe knowledge that is uncertain and ambiguous. With this advantage, FTL techniques have also been researched and applied to many different problems, from classification to regression, in many fields, such as image recognition [29], medicine [39], [40] [15], education, natural language processing [29], finance [41], [26], smart environment [27], etc.

The aim of FTL is to transfer existing knowledge in an uncertain, ambiguous environment. FTL is proposed based on combining transfer learning methods with fuzzy theory. Therefore, fuzzy transfer learning research can also be classified as transfer learning, such as inductive transfer, unsupervised transfer, feature transfer, etc., like traditional transfer learning. However, transfer learning and fuzzy set theory are powerfully combined in a direction based on fuzzy inference systems to create fuzzy inference systems for the target domain in cases of data information deficiency [26, 28, 42, 43]. Therefore, fuzzy transfer classification can be based on inference models.

1.2.3 Sampling and sampling methods

Popular machine learning methods are learning from data; "good" data will bring good performance to machine learning models [38]. However, collecting complete data for the learning process is impossible in terms of time and cost. Therefore, research on sampling methods—retrieving data with smaller sizes, characteristics, and noise reduction—is one of the research issues of interest [44–47].

Sampling methods are divided into probability sampling and non-probability sampling [48, 49]. Probability sampling (PS) is a method in which individuals are randomly selected; each individual in the research population has an equal chance of being selected and does not depend on the subjective opinion of the researcher. Meanwhile, non-probability sampling (NPS) means the researcher intentionally selects subjects to participate in the research based on available individuals when collecting data and does not calculate sample size. Non-probability sampling can be convenience sampling, target sampling, or purposive sampling to probe or deeply understand a problem in a population (knowledge, attitudes, beliefs, etc.). Sampling methods must be systematic and determined to draw valid inferences from the sample.

1.3 Limitations of complex fuzzy inference systems and research problems

CFIS effectively solves inference problems with uncertain, ambiguous, cyclical, periodic data. Applications of CFIS, such as decision-making, prediction, forecasting, etc., have made contributions in the fields of research and knowledge processing [2, 5–7]. However, the process of considering phase components in the data and the rule domain makes the execution time of these inference models consume considerable time. In reality, building complex fuzzy inference models on large data sets and continuously updating the time needed to update the inference system is very time-consuming. This creates limitations in the widespread application of complex fuzzy inference models to actual systems, especially systems that require re-updating or inference in a short period.

Meanwhile, one of the capabilities of TL is to reduce learning time by reusing knowledge from related source domains. However, most previous proposals have yet to pay attention to this, mainly only concerned about the problem of knowledge deficiency in the target domain [11]. With that analysis, the thesis proposes research on transfer learning techniques to reduce the time to create a CFIS for the target domain based on the available source domain.

Research problem

The context for the research is that building an M-CFIS system for the target domain with size N_ψ (very large) in a shorter period than traditional construction methods is necessary. By taking advantage of the M-CFIS system of another domain, I called the source domain, which has an available size of N_O . The target domain and the source domain have a relationship with each other. The research problem can be described as Figure 1.1.

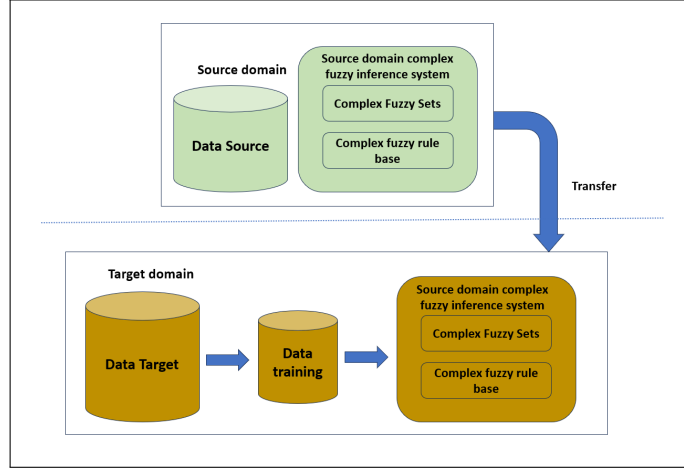


Figure 1.1: Transfer learning problem model

Figure 1.1 shows that the specific goals that need to be researched and resolved here are:

1. Use transfer learning techniques to transfer knowledge from the source domain (complex fuzzy sets, complex fuzzy rule bases) to the target domain.
2. Combine with a small amount of data selected from the target domain.

To obtain the M-CFIS inference system for the target domain in less time than building the target domain inference system directly from the entire target domain data.

1.4 Theoretical basis

1.4.1 Complex fuzzy sets

Definition 1.1. [1] A pair $(l, \mu_H(l))$ is defined as a complex fuzzy set on the universal space U , where the function $\mu_H(l)$ is a complex-valued membership function for any $l \in U$ with the following form:

$$\mu_H(l) = \vartheta_H(l) \cdot e^{j\varpi_H(l)} \quad (1.1)$$

with $j = \sqrt{-1}$, amplitude component $\vartheta_H(l) \in [0, 1]$ and phase component $\varpi_H(l) \in (0, 2\pi]$.

1.4.2 Mamdani complex fuzzy inference system [5]

The Mamdani CFIS inference system is developed based on complex fuzzy set theory and the Mamdani inference system, including the following inference steps:

Let $l_1, l_2, \dots, l_m \in \mathbb{C}$ be the input data set.

Step 1: Initialize complex fuzzy rules (CFRs)

CFR_1 : If $l_{1,1}$ is $H_{1,1}$ $\mathbf{O}_{1,1}$ $l_{1,2}$ is $H_{1,2}$ $\mathbf{O}_{1,2}$ \dots \mathbf{O}_{1,m_1-1} l_{1,m_1} is H_{1,m_1} then Z_1

CFR_2 : If $l_{2,1}$ is $H_{2,1}$ $\mathbf{O}_{2,1}$ $l_{2,2}$ is $H_{2,2}$ $\mathbf{O}_{2,2}$ \dots \mathbf{O}_{2,m_2-1} l_{2,m_2} is H_{2,m_2} then Z_2

...

CFR_k : If $l_{q,1}$ is $H_{q,1}$ $\mathbf{O}_{q,1}$ $l_{q,2}$ is $H_{q,2}$ $\mathbf{O}_{q,2}$ \dots \mathbf{O}_{k,m_k-1} l_{k,m_k} is H_{k,m_k} then Z_t

For all u, v satisfy:

(1) $(u, v) \in \{1, 2, \dots, m\}$, $\forall i$ $1 \leq u, 1 < u, 2 < \dots < u, m_u \leq m$

(2) $\mu_{H_{u,v}}(l_{u,v}) = \vartheta_{H_{u,v}}(l_{u,v}) e^{j\varpi_{H_{u,v}}(l_{u,v})}$, with $\vartheta_{H_{u,v}} : \mathbb{C} \rightarrow [0, 1]$ and $\varpi_{H_{u,v}} : \mathbb{C} \rightarrow (0, 2\pi]$

- (3) $\mu_{Z_u}(w) = \vartheta_{Z_u}(w)e^{jZ_u(w)}$, with $\vartheta_{Z_u} : \mathbb{C} \rightarrow [0, 1]$ and $\varpi_{Z_u} : \mathbb{C} \rightarrow (0, 2\pi]$
 (iv) $O_{u,v} =$ and if $N_{u,v} = T_0$, T_0 is a T - standard
 (v) $O_{u,v} =$ or if $N_{u,v} = S_0$, S_0 is a T - standard reference.

Step 2: Fuzzy complexifies the input data - using complex fuzzy membership functions

$$\mu_H(l) = \vartheta_H(l) e^{j\varpi_H(l)} \quad (1.2)$$

Step 3: Calculate the strength of the rule.

Determine the strength ϖ_u of each rule according to the formula: $\varpi_u = \tau_u e^{j\xi}$

Step 4: Calculate the outputs of each CFR

The value of the conclusion of the CFR is calculated using the Mamdani entailment rule.

Choose the function $U_0 : [0, 1]^2 \rightarrow [0, 1]$ with $U_0(1, 1) = 1$, and the function $g_0 : (0, 2\pi]^2 \rightarrow (0, 2\pi]$ with $g_0(2\pi, 2\pi) = 2\pi$.

The form of the corresponding output function for each complex fuzzy rule CFR_p is calculated by the formula: $\Gamma_p(z) = U_0(\tau_p, r_{C_p}(z)) e^{ig_0(\psi_p, \varpi_{C_p}(z))}$.

Step 5: A summary of the output results of sophisticated fuzzy rules The following is the procedure for synthesizing output results:

$$D(w) = \Gamma_1(w) + \Gamma_2(w) + \dots + \Gamma_q(w). \quad (1.3)$$

With $D = F(\mathbb{C}, \mathbb{C})$

Step 6: Complex defuzzification

Function $\phi : F(\mathbb{C}, \mathbb{C}) \rightarrow \mathbb{C}$, gives the output value according to: $y_{op} = \phi(D)$

1.4.3 Transfer Learning

TL [38] has been proposed in many studies to transfer knowledge between domains, shortening the knowledge gap of the target domain based on related source domain knowledge, and is a promising machine learning method to solve problems encountered in practice. When considering transfer learning, people mention issues such as transfer domain, source and target domain tasks, transfer techniques, etc. These factors will determine the methods and techniques of transfer learning.

Definition 1.2. [38] *The domain is defined by $\zeta = \{G, T(L)\}$, where G represents the domain characteristic and $T(L)$ describes the probability distribution of the element $L = \{l_1, \dots, l_m \in G\}$.*

Definition 1.3. [38] *A task E is given by $E = \{W, \varrho\}$, with a label space W and a prediction function ϱ , which is an implicit function that can be learned from sample data.*

The output is the machine learning predicted conditional distribution labels:

$$\varrho(l_i) = \{T(w_k | l_i) | w_k \in W; k = 1, \dots, |W|\} \quad (1.4)$$

In reality, a domain includes both labeled and unlabeled data. For example, given a source domain ζ_O and a source task E_O the source domain is often observed through the pair (element, label) is: $\zeta_O = \{(l, w) | l_i \in G_O; w_k \in W_O; i = 1, \dots, m_{iO}; \text{ and } k = 1, \dots, m_{kO}\}$. Include several data samples with or without labels along with the target domain. In reality, a domain consists of both labeled

and unlabeled data. For example, given a source domain ζ_O and a source task E_O , the source domain is often observed through the pair (element, label) is: $\zeta_O = \{(l, w) | l_i \in G_O\}$; $w_k \in W_O$; $i = 1, \dots, m_{iO}$; and $k = 1, \dots, m_{kO}$. Include several data samples with or without labels along with the target domain.

Definition 1.4. [38] *Let ζ_O , E_O , ζ_ψ and E_ψ is the source domain and source task, target domain and target task respectively. Transfer learning uses the knowledge obtained from the source domain to reuse it for the prediction function ρ in the target domain ζ_ψ where $\zeta_O \neq \zeta_\psi$ or $E_O \neq E_\psi$.*

1.5 Experimental

This thesis's experimental data sets include 4 data sets from the UCI standard data warehouse and the hepatitis data set collected from the Thai Nguyen Iron and Steel Hospital (Table ref:1.2). Selecting these data sets based on criteria such as collection time or additional components in the data ensures the role of the phase component of the data. This emphasizes the importance of analyzing specific and complementary components for each attribute.

Table 1.1: List of experimental data sets

No.	Dataset	Number of attributes	Number of records	Label
1	BreastCancer	9	680	2
2	Diabetes	5	390	2
3	Creditcard	16	8636	7
4	Liver	9	4156	2

For each input record, with each attribute value, generating real and imaginary parts for the data is done according to the following principle: The real part is taken from the original input value of the attribute. The imaginary part is calculated by summing the column-wise and row-wise variances ($\text{var.R}(\text{record}) + \text{var.A}(\text{attribute})$) [50]. The result of this process is a new data set. The original data set is called real data, while the newly created data set is called virtual data. These two data sets will be used as input data for further experiments.

1.6 Conclusion of chapter 1

Within the framework of Chapter 1, the thesis has presented related research on inference and complex fuzzy inference systems, focusing on the Mamdani complex fuzzy inference system for solving inference problems based on information. The information is uncertain, ambiguous, and cyclical. At the same time, the thesis also points out the limitations of the current research direction.

Also, in this chapter, the thesis presents transfer learning techniques, transfer learning methods, and fuzzy transfer learning methods. With the characteristics of transfer learning techniques, they can solve the problem of knowledge deficiency and shorten learning time. This foundational knowledge will be used in the following chapters of the thesis to propose models to address the stated research gaps.

The results of this research are published in works [1, 2] of the thesis.

Chapter 2

TRANSFER LEARNING MODEL ON COMPLEX Fuzzier INFERENCE SYSTEM

To solve the research problem in Chapter 1, this thesis chapter proposes a transfer learning model based on a complex fuzzy inference system. As follows:

- Proposing the CFTL model, based on FTL and M-CFIS, to shorten the time to build the Mamdani complex fuzzy inference system.
- Introducing the CFTL model includes four steps: adjusting the source domain, selecting subsets in the target domain, adapting the rule, and combining rules for the final set of adaptive rules.
- Proposing a new definition of adaptation rules and candidate rules to select the essential rules in the rule base to adapt the rules to the target domain data.
- Verify the ability of the CFTL model to handle knowledge transfer situations with information-deficient contexts in the target domain through the implementation of test scenarios.
- Demonstrate the performance of the proposed new model through experiments performed on data from UCI [51] as well as real data sets, evaluating the accuracy, number of rules, and execution time.

The essence of the proposed CFTL model lies in the inference mechanism according to the Mamdani complex fuzzy inference model. The new point is that the CFTL system is supplemented with a fuzzy transfer learning feature. Besides, according to the researcher, most current CFIS systems do not take advantage of the phase factor. In contrast, the proposed CFTL model performs calculations on amplitude and phase parts during the decision-making process.

2.1 Some concepts and definitions

In the theory of transfer learning, the similarity relationship between source and target domain data plays an important role. It helps determine the possibility of negative transfer and is the basis for deciding transfer methods and techniques. Researchers can use similar factors to build transfer learning techniques to gain time advantages. In the context of the relationship between the source and target domains, this study shows that the source and target domains have the same number of attributes and corresponding output tasks but are different in data distribution. This answers the proposed model's question, "When will the transfer occur?"

Definition 2.1. *Transfer learning on complex fuzzy inference systems*

Let ζ_O , E_O , ζ_ψ and E_ψ is the source domain and source task, target domain and target task respectively. Transfer learning on complex fuzzy inference systems uses the knowledge obtained from the source domain, reused for the prediction function ϱ in the target domain ζ_ψ . With source and target domain constraints as follows:

- ζ_O, ζ_ψ are the same in distribution and number of attributes but different in the range of values of the attributes.
- $E_O = E_\psi$, same mission.

Definition 2.2. *Similarity between two domains for CFTL.*

Assuming ζ_U and ζ_V are two domains, the similarity of the domain ζ_V to the domain ζ_U is determined by the following formula:

$$DI(\zeta_U, \zeta_V) = \frac{\sum_{i=1}^m S_i}{m} \quad (2.1)$$

Where:

$$S_i = \left\{ \begin{array}{l} 1, \text{ if } x_i^V \geq L^U \text{ and } x_i^V \leq R^U \\ \frac{1}{|L^U - x_i^V|}, \text{ if } x_i^V < L^U \\ \frac{1}{|x_i^V - R^U|}, \text{ if } x_i^V > R^U \end{array} \right\} \quad (2.2)$$

with x_i^V is the value of the i th element of the domain ζ_V , $L^U = \min_{j=1,n} x_j^U$; and $R^U = \max_{j=1,n} x_j^U$; m, n are the number of data lines in ζ_V, ζ_U respectively.

Lemma 1: With similarity DI between two regions ζ_U and ζ_V , the value DI satisfies satisfy the following requirements:

1. $DI(\zeta_U, \zeta_V) \in [0, 1]$
2. $DI(\zeta_U, \zeta_V) = 1, IF \zeta_V \subseteq \zeta_U$
3. $DI(\zeta_U, \zeta_V) \rightarrow 0 IF \forall x_i^V \ll L^U \text{ or } \forall x_i^V \gg R^U$

To consider the rules transferred to the target domain, a definition of suitable rules (including adaptive rules and candidate rules) is presented. But first, this determination is based on the inference mechanism of the M-CFIS system and the robustness assessment process proposed below (Definition 2.3).

Definition 2.3. *Strength of complex fuzzy rules for transfer rule selection*

The following function determines the strength of the complex fuzzy rule:

$$\omega_u = \min_{k=1,n} \left(\sqrt{(F_A \cdot \cos(F_P))^2 + (F_A \cdot \sin(F_P))^2} \right) \quad (2.3)$$

where $k \in \overline{1, n}$, $F_A = \vartheta_H(l_{u_k})$ is the amplitude strength, and $F_P = w_H(l_{u_k})$ phase strength.

Definition 2.4. *Adaptive rules and candidate rules are defined as follows:*

(i) *Complex fuzzy rule (CFR) is an adaptive rule on data t when:*

- *The rule's output is the same as the output of the t th data stream.*
- *The flammability of the rule is greater than the threshold value ε*

(ii) *A complex fuzzy rule is called a candidate rule on data stream t if it satisfies:*

- *The output of the rule is the same as the label of data stream t*
- *Each strength amplitude that $(F_A \geq \varepsilon)$ is $\frac{\text{count}(F_A \geq \varepsilon)}{\text{count}(F_A)} \geq \alpha$*
- *Every strength amplitude that $(F_A < \varepsilon)$ is $(F_P \geq \beta)$*

with $\varepsilon, \alpha, \beta \in [0, 1]$: predefined parameters.

2.2 Complex fuzzy transfer learning model

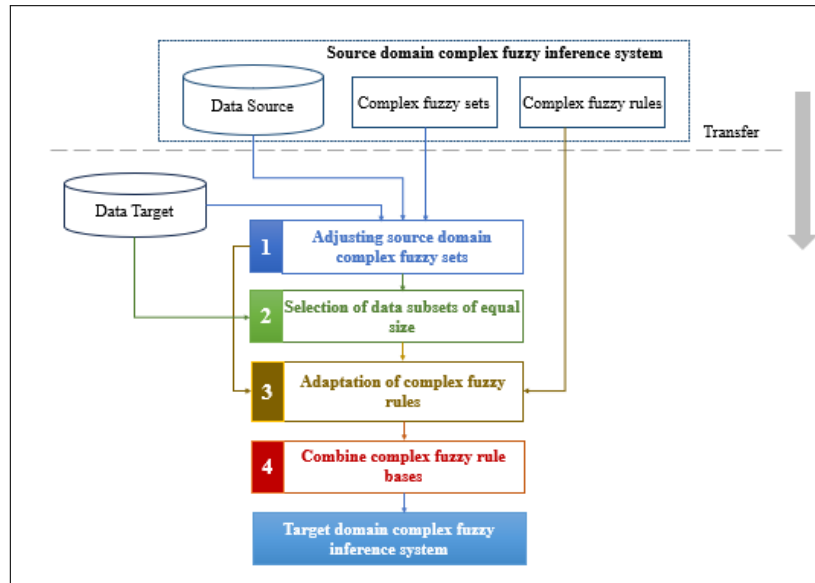


Figure 2.1: Transfer learning model on complex fuzzy inference system

The proposed CFTL model is designed as a knowledge transfer model in which the source and target domains have a transfer relationship (Definition 2.1). In this model, the source and target domain tasks have the same size but have different distributions. CFTL attempts to bridge the gap of distributional differences across the learning process and adapt learning from one context to another. Context change can be due to domain change, lack of information, situation, etc. CFTL is proposed to transfer knowledge from one space to another to reduce processing time and generate a new rule base with acceptable prediction quality.

The design of CFTL (Figure 2.1) includes four stages. First, the domain range adjustment phase changes the CFS information in the source domain to match the target domain data range. Next, a procedure is introduced to select $D_{sub}(k)$ subsets based on data labels and attribute fields at the destination. Then, each data record in these subsets $D_{sub}(k)$ is used to adapt the rules. Finally, the adaptive CFRs are combined to create the final adaptive rule set - $Rad(Final)$, the complex fuzzy rule set used for inference in the target domain.

2.2.1 Adjust source domain complex fuzzy sets

Since the data domain ranges differ in the source and target domains, this section provides an adjustment method to transfer fuzzy sets from the source domain to the target domain. To adjust the source input domains to match the input domains at the destination, we must change the source input intervals to the destination input intervals. The amplitude and phase components between the source and target domain properties are compared. The range boundaries of source domain attributes will expand or contract due to data from attribute ranges in the target domain. Each amplitude and phase component in the target domain input is compared concerning the amplitude and phase intervals in the source domain. If it is smaller than the left margin, then the left margin is narrowed; otherwise, the right boundary is expanded if it is larger than the right boundary. The result is a set of new hubs adapted to the target domain input.

2.2.2 Select subsets of data

Selecting subsets of the target data is essential to reducing the useless information of the target samples required in the rule adaptation process. Each selected Dsub subset of size K is separate and distinct. The process of choosing Dsub subsets can be described as follows: With data in the target domain, we need to select N_d ($N_d = \eta * N_\psi$, η is the learning rate) the record is divided into N_{Dsub} ($N_{Dsub} = N_d / K$) data subsets of size K . Selected records are random, not reselected.

2.2.3 Adaptive complex fuzzy rules

Each selected Dsub subset will be used to create a set of adaptive rules (called R_{ad}) by adjusting complex fuzzy rules. CFR adjustment is based on the inference mechanism of M-CFIS, using the CFR of the source domain as the initialization rule base for each R_{ad} . This rule base is tested iteratively to find the most adaptive rules with greater strength. This helps improve the target domain. Each data record t in the subset Dsub will create an adaptation on the set of complex fuzzy rules R_{ad} , that is, develop a process of editing or adding new rules to adapt to data record t . The parameter *Maxfire* is called the learning rate. The number of adaptation rules for each record depends on this parameter.

With the proposed model, a process randomly selects N_d data instead of taking the entire target domain data to generate rules. That means that these N_d data records will represent N_ψ data records in the target domain. Suppose a data record has generated a rule, then for N_ψ of data, generate N_ψ of rules. Meanwhile, N_d is used to represent N_ψ to help create the target domain rule base. Therefore, with data N_d after maximum adaptation, N_ψ rules will be generated, and each record will generate N_ψ / N_d law. This is the basic idea for determining *Maxfire* when adapting the rule base.

2.2.4 Combining complex fuzzy rule bases

After obtaining the set of adaptive CFRs $\{\text{Rad}(1), \text{Rad}(2), \dots, \text{Rad}(k)\}$, they will be synthesized by eliminating similar CFRs and weak CFRs to obtain the set of final adaptive CFRs (named Rad (Final)) as the set of CFRs for the target data.

2.3 Experimental

2.3.1 Experimental scenario

to evaluate the performance of CFTL, it is used to create a complex fuzzy inference model for the target domain on a small portion of data instead of using the entire target domain. The experimental scenario on CFTL is implemented as follows:

- From an initial data set, after randomly separating an amount of test data (20%), the remaining data (80%) continue to separate a part considered the source domain and a part considered the target domain. To ensure the context is similar to the problem posed (chapter 1), in the experiment, only a small amount of data (10%) is taken as the source domain, and the remaining (90%) is the target domain. Separating data considered the source domain is done using a simple random sampling method to ensure that the source and target domain data have the same distribution but may differ in the range values of the attributes.

- While performing transfer learning on the CFTL model, an amount of target domain data continues to be taken for the rule adaptation process according to the learning rate (20%); this part of the data is taken randomly naturally divided into Dsub sets for the transfer learning process in the proposed model.

The CFTL model is compared with the traditional M-CFIS model method (where all 80% of the data will be used to generate rules for the target domain inference system). The experimental results are evaluated based on three indicators: calculation time, accuracy, and number of rules.

In summary, experimental results on both types of data, standard and real data, show the effectiveness and significance of the execution time of CFTL.

2.4 Conclusion of Chapter 2

This chapter proposes a new FTL model on the Mamdani CFIS system to reduce the model-building time for the target domain through learning knowledge from related domains. The proposed system selected data from the target domain to adjust the source domain rule base according to the mechanism of the Mamdani CFIS inference system. The data size is chosen to be much smaller than the entire data in the target domain to reduce the computation time. The fuzzy transfer learning technique has been applied to transfer the source domain complex fuzzy inference system to the target domain through source domain adaptive adjustment and complex fuzzy rule adaptation stages.

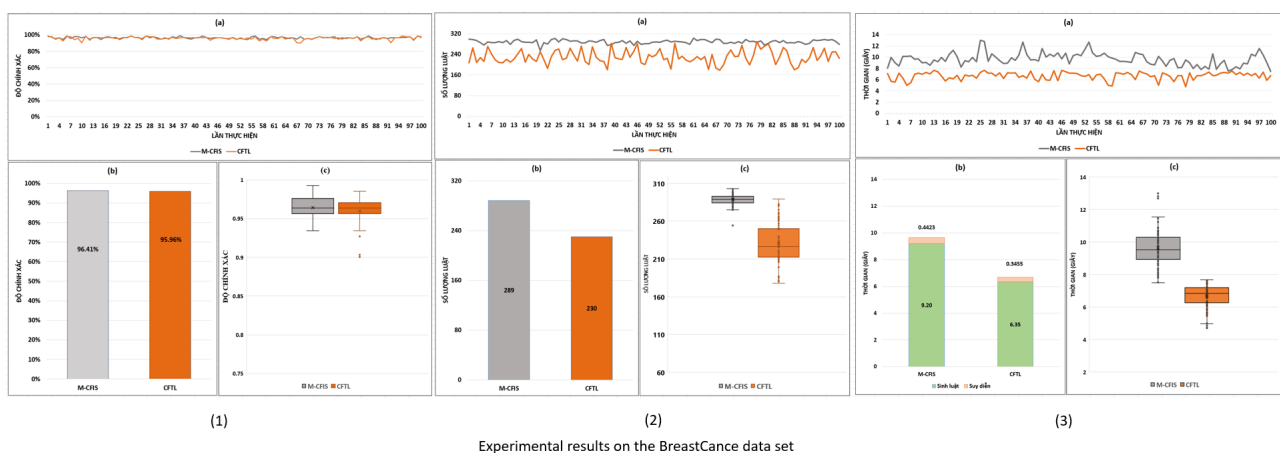
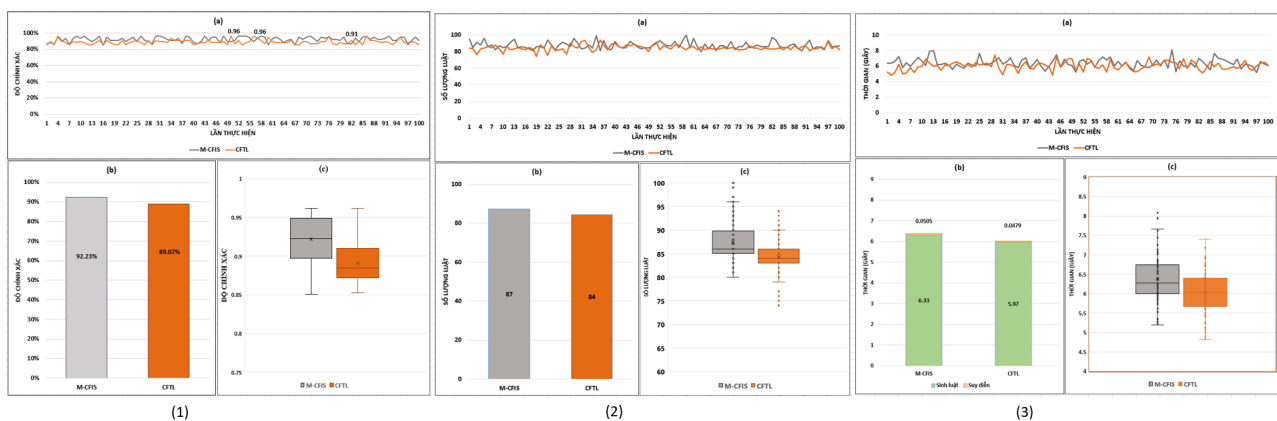


Figure 2.2: Experimental results on Diabetes and Breast-Cancer sets

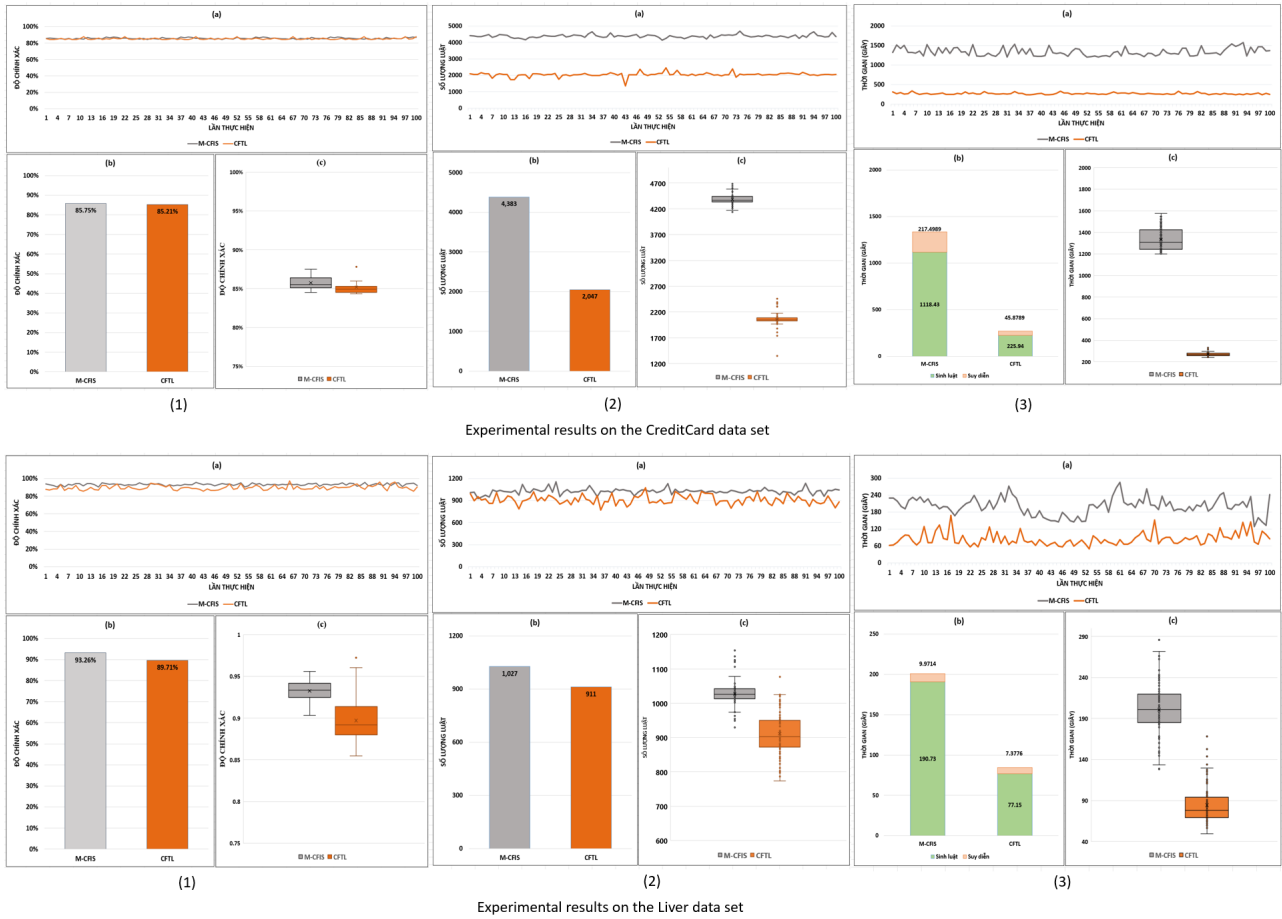


Figure 2.3: Experimental results on CreditCard and Liver

Experimental results on the UCI data set and real data sets show that CFTL can bring the desired results in situations where it is necessary to build a target domain CFIS (with relatively large data) quickly by reusing the complex fuzzy inference system of an existing similar domain (source domain).

The research results are published in the thesis work [3].

The proposed model has shortened the time to create the Mamdani CFIS system for the target domain thanks to the ability to transfer rules combined with limited data in the target domain. However, the proposed CFTL transfer learning model does not use this phase component in editing rules. Besides, although the time to create a complex fuzzy rule base for the target domain's complex fuzzy inference system has been shortened, the complex fuzzy inference time has yet to be considered. These are the limitations of the proposed CFTL model and the research issues raised for further research in Chapter 3.

Chapter 3

COMPLEX FUZZY TRANSFER LEARNING SYSTEM BASED ON CFRG STRUCTURE

Reducing inference time on fuzzy inference systems can be done by many methods, such as optimizing inference rules, using relationship diagrams, dividing the system, etc. Reducing the inference time on the complex fuzzy inference system is necessary because the phase component has significantly increased the inference time of the complex fuzzy inference system. Besides, reducing the inference

time on the complex fuzzy inference system also reduces the time for the CFTL transfer learning model proposed in chapter ?? because the rule adaptation process of the CFTL model is built on the complex fuzzy inference mechanism. In addition, a mechanism to suggest a more manageable selection of amplitude and phase values when editing rules during the rule adaptation phase will also help reduce the time spent searching for new amplitude or phase values. Overall, it also helps the transfer learning time on the complex fuzzy inference system to be further improved in terms of time.

The tree data structure is always suitable for representing data with hierarchical relationships, making it easy for quick data retrieval. Especially decision trees - one of the famous techniques to help classify data [52–54] and its variations such as RandomForest [55, 56], Gradient Boosting Trees [57], fuzzy decision tree (FDT) [54, 58]. In FDT, fuzziness is applied to data and decisions, enabling it to work more effectively in situations where data is not always clear and precise [59? – 66]. Tree data structures have also been integrated into transfer learning methods [11, 67, 68]. This combination produced excellent computational complexity and interpretability [69], [70].

Another structure that is also effective in representing data and knowledge is a graph. The Directed Acyclic Graph (DAG) is an important data structure in computer science and other fields. A DAG is a type of graph whose vertices and edges indicate the direction of movement from one vertex to another, and no cycles exist in the graph [71]. DAG is present in many studies in different research fields such as medical [72, 73], risk prediction [74] etc. DAG’s simple, directed, acyclic structure meets data analysis needs, processes, quick access, etc.

Continuing the research from Chapter 2, this chapter proposes to use the CFRG structure (based on DAG and tree structures) to represent complex fuzzy rules for:

- A visual representation of complex fuzzy rules, including amplitude and phase separation on the CFRG structure.
- Proposing some algorithms on CFRG structure.
- Improves the performance of the time-complex fuzzy inference process.
- Applying the CFRG structure to improve the CFTL model proposed in Chapter 2.

To build the CFRG structure, some concepts and definitions are introduced as follows.

3.1 Some concepts and definitions

To define a CFRG structure to represent a set of complex fuzzy rules, including amplitude values and phase values, and meet the stated purposes, a complex fuzzy node structure is introduced in Definition ??.

Definition 3.1. *Complex fuzzy node (CFN)*

A complex fuzzy node CFN is a structure of 3 parts (p_A, p_P, p_l) . In there:

- *The first part, p_A is a pair $(aValue, afreq)$ where $aValue$ is the amplitude linguistic variable and $afreq$ the frequency of its occurrence on the same attribute.*
- *The second part, p_P is a list of pairs $(pValue, pfreq)$, each pair includes the phase linguistic variable $pValue$ and its frequency of occurrence on the same attribute called $pfreq$.*

- The third part, p_l , is a list of links to connect to child nodes in order of law and phase.

From the above definition of complex fuzzy node, the definition of CFRG structure is stated as follows:

Definition 3.2. *CFRG (Complex fuzzy rule Graph - CFRG) structure A CFRG structure is a tree-like structure representing a set of complex fuzzy rules. In there:*

- Each node on the CFRG is a CFN that represents the properties of one or more rules with the same amplitude language variable but may differ in phase.
- Each edge represents a T-norm operator (AND or OR).
- The phase parts of the node belonging to the same rule will have the same index.

In addition to the two definitions of complex fuzzy nodes and CFRG structure, several definitions to measure the degree of difference between two CFRGs and evaluate the strength of CFRG are also introduced.

Definition 3.3. *Measure of difference between two structures CFRG Let two structures CFRG_1 and CFRG_2 be extracted from a certain parent structure CFRG. The difference between CFRG_1 and CFRG_2 is calculated as follows:*

$$DI = \sum_i \frac{DI_i(CFRG_1, CFRG_2)}{h * (n(CFRG_1.CFN) + n(CFRG_2.CFN))_i}; \quad (3.1)$$

With:

- $DI_i(CFRG_1, CFRG_2) = \sum_j |CFRG_1.CFN_j.freq - CFRG_2.CFN_j.freq|_i$, where j is the language variable. - $i \in [1, h)$, h maximum level on CFRG structure;
- $n(CFRG_1.CFN), n(CFRG_2.CFN)$ is the total frequency of nodes on the same level of the structure CFRG_1, CFRG_2 respectively.

The node strength on the CFRG structure is presented below to measure its strength.

Definition 3.4. *The strength of the complex fuzzy node*

The strength of node CFN at level k is denoted by (S_{CFN_k}) , given by the formula:

$$S_{CFN_k} = \sqrt{(SA_{CFN_k} \cdot \cos(SP_{CFN_k}))^2 + (SA_{CFN_k} \cdot \sin(SP_{CFN_k}))^2} \quad (3.2)$$

With:

$$SA_{CFN_k} = \mu(CFN_k.aValue) + \left(1 - \mu(CFN_k.aValue) \frac{CFN_k.afreq}{n(CFN_k)}\right) \quad (3.3)$$

$$SP_{CFN_k} = \max_{i=1, n} \left(\omega(CFN_k.pValue_i) + \left(1 - \omega(CFN_k.pValue_i) \frac{CFN_k.pValue_i.pfreq}{CFN_k.afreq}\right) \right) \quad (3.4)$$

In there:

- SA_{CFN_k}, SP_{CFN_k} are the amplitude strength and phase strength of the node.
- $\mu(CFN_k.aValue)$ amplitude part, $\omega(CFN_k.pValue_i)$ phase part of each end into the law.
- n number of phase parts of the node.
- $n(CFN_k)$ is the total frequency of the nodes at the k^t h level.

3.2 Algorithms on CFRG structure

To apply the CFRG structure to improve the complex fuzzy inference process's performance, adapting the CFTL model rules proposed in Chapter ???. Some algorithms below that allow manipulation of the CFRG structure are introduced as follows:

- The algorithm adds a rule to the CFRG.
- Complex fuzzy rule review algorithm on CFRG.
- Algorithm to search for a rule on CFRG.
- Algorithm to delete a rule from CFRG.
- Complex fuzzy inference algorithm on CFRG.
- Algorithm for editing rules on CFRG.
- Algorithm to separate child CFRG from parent CFRG.
- Algorithm for mixing two CFRG structures.

3.3 Complex fuzzy transfer learning model based on CFRG structure (CFRGTL)

To increase the performance of the transfer learning model proposed in Chapter ??, a complex fuzzy rule representation of the fuzzy rule CFRG (Definition 3.2) is introduced. This fundamentally changes the operations in each stage of the transfer learning model.

Compared to the previous model (Figure 2.1), the new model proposed in this section has added several important stages (Figure 3.1):

- Initialize CFRGs from source domain CFRs;
- Separate the child CFRG structure from the parent CFRG structure;
- Adaptation on CFRG Structures;
- Finally, merge the post-modified CFRG structures.

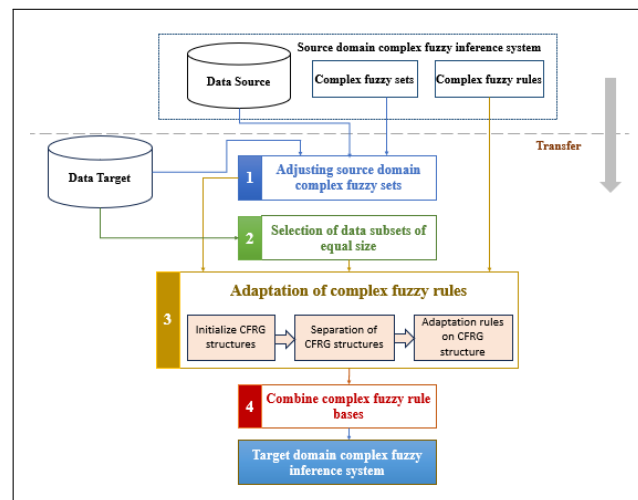


Figure 3.1: Complex fuzzy transfer learning model based on CFRG structure (CFRGTL)

3.4 Experimental

To examine the capabilities of the CFRG structure proposed above, two experimental scenarios are presented: 1. Experimentally evaluate inference time on CFRG compared to traditional Mamdani - CFIS; 2. They experimentally assess the applicability of CFRG by evaluating the performance of the complex fuzzy transfer learning model based on the CFRG structure (CFRGTL).

3.4.1 First experimental scenario

To demonstrate the ability to make fast inferences on CFRG, the 4 data sets in Table 1.1 are generated using the traditional M-CFIS method [7]. From the generated set of complex fuzzy rules, I perform complex fuzzy inference in two ways: method 1, inference on regular arrays; Method 2, the structure is based on CFRG and inference on CFRG. The results are shown in the table below (Table 3.1).

Table 3.1: Table of experimental results according to scenario 1

No	Datasets	Number of records	Number of rules	DataTest	M-CFIS Inference Time	CFRGTL Inference Time
1	Diabetes	390	101	78	0.1099	0.0786
2	Breast-Cancer	683	288	137	0.7483	0.5821
3	CreditCard	8636	4419	1727	240.223	126.243
4	Liver	4156	1044	831	23.4976	11.7312

The experimental results in table 3.1 show that the rule structure on CFRG allows faster inference than the structure on a conventional two-dimensional array (M-CFIS). With two smaller data sets, Diabetes and BreastCancer, inference time on CFRG is about 27% - 28% faster than regular M-CFIS. However, with the two data sets Credit Card and Liver with larger sizes (both in the number of records and attributes), the inference time has been dramatically reduced by 45% - 50%. This shows CFRG's ability to handle large datasets.

3.4.2 Second experimental version

To evaluate the applicability of CFRG, the proposed CFRGTL is further tested on UCI data sets and real data (Table 1.1) and compared with related methods.

The results of four data sets corresponding to two test cases are presented in Figure 3.2 and Figure 3.3, respectively.

Comment

First, structuring the law on the CFRG structure - an unprecedented representation helps the law review process on the CFRG faster. It also makes the complex fuzzy inference process on CFRG quicker.

Second, selecting a new amplitude according to the phase proposal helps reduce amplitude selection time in the rule editing phase.

Third, separating each child CFRG from the original CFRG and implementing multi-threaded techniques for the editing inference process also helps shorten the time compared to the model proposed in chapter 2.

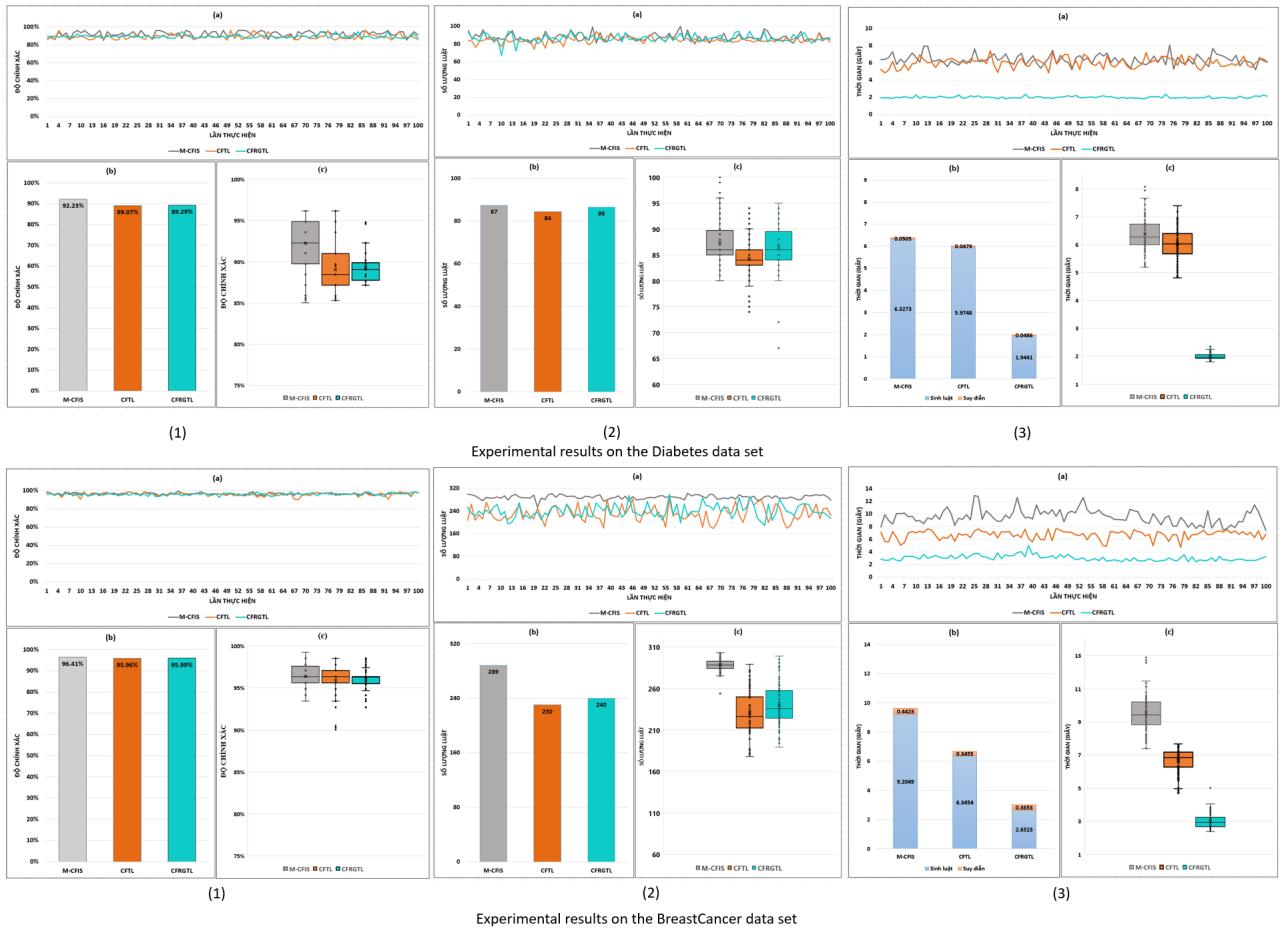


Figure 3.2: Experimental results on Diabetes and Breast-Cancer datasets

3.5 Conclusion of Chapter 3

In this chapter, the author proposes a new CFRG structure that allows storing both the amplitude part, phase part, and frequency part on each node. The advantage of this storage method is that it helps visualize the phase factor of each rule node and more clearly see their influence on the rule during the inference or editing process.

The superiority of the CFRG structure has been clearly shown in the first experimental results. When used for complex fuzzy inference, it has shortened the output inference time of M-CFIS systems. In addition, using the CFRG structure for the rule adaptation process in complex fuzzy transfer learning also shortens the adaptation time due to two factors: 1) selecting a new correction value based on the phase portion on each node; 2) multithreading is possible through splitting CFRG substructures.

The results of this research are published in the work [6] of the thesis.

Also, from the research results of this chapter, several further research questions are raised: 1) Can the CFRG structure support better law synthesis, increasing accuracy or reducing the number of rules? 2) Can the context of the transfer learning model be extended to other cases? 3) Can other stages of the original transfer learning model in Chapter 2 continue to develop to increase the performance of the model? This is also the goal of developing the proposed model in subsequent studies.

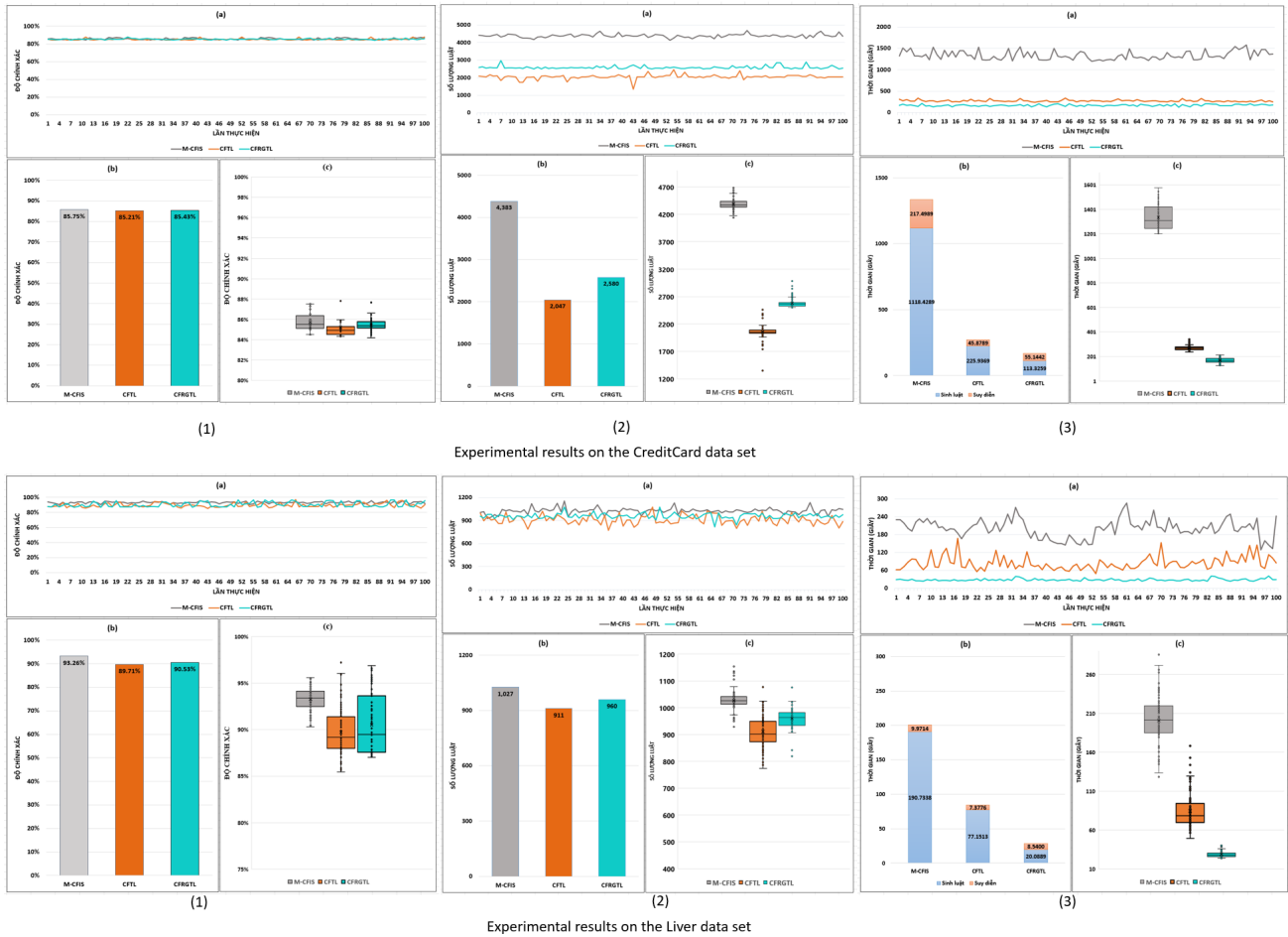


Figure 3.3: Experimental results on CreditCard and Liver datasets

CONCLUSION

Main results of the thesis

The research objective is to focus on researching and proposing the construction of a transfer learning model on a complex fuzzy inference system to improve the learning time (time to build the inference system) for the target domain.

- **First**, the thesis has proposed a transfer learning model based on the Mamdani complex fuzzy inference system.
 - The proposed method used transfer learning techniques in building the Mamdani complex fuzzy inference system for the target domain to shorten the time compared to the previous method.
- **Second**, the thesis also proposed a way to represent complex fuzzy rule sets on the CFRTrie structure:
 - Visually represent complex fuzzy rules on the CFRTrie structure.
 - Improve complex fuzzy inference time of Mamdani complex fuzzy inference models. On that basis, continue to improve the rule adaptation time (marking adaptive rules, candidates, and editing value selection) of the previously proposed transfer learning model.

The thesis again demonstrates the ability of transfer learning techniques to shorten learning time. At the same time, it also confirms the role of additional elements in the data for learning and inference tasks. As in this case, the Phase

component.

Some limitations

Besides the research results achieved, the research in the thesis still has some limitations, such as:

- The thesis has only experimented on digital data sets, but spatial data has not been implemented yet.
- The steps in the proposed fuzzy transfer learning model are still simple.
- The context for the transfer learning model still has many constraints, and the conditions are too ideal.
- The capabilities of the proposed CFRG structure have not been fully exploited, such as the ability to handle missing data, the ability to synthesize and optimize rule sets on the structure, etc.

Development direction of the thesis

In the future, further development directions for the thesis can be carried out in the following research directions:

- Testing algorithms on big data models
- Adding more dimensions of the data to demonstrate the superiority of the model;
- Continue testing the models proposed in the thesis with more complex data sets in different areas of life, such as health, economics, geography, etc.
- Apply, deploy, and integrate proposed research for real-life systems such as weather forecasting, natural disasters, storm forecasting, etc.;