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RESEARCH ON DEVELOPING GROUP RECOMMENDER SYSTEM BASED ON APPROACH OF INTUITIONISTIC FUZZY SET AND CHOQUET INTEGRAL

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INTRODUCTION

1. Problem statement (Necessity of the research problem)

In recent years, Recommender Systems (RS) are considered as an information filtering mechanism for users when information systems (IS) have too much data and the way to search for data by keywords is not really suitable. In fact, RS has been researched, developed and applied in most of the ISs with a large number of users today [1]. In which, the recommender system aims to solve the problem of giving appropriate recommendations to a group of users, called Group Recommender Systems (GRS) [2]. In terms of modeling, GRS is a general model of a single-user RS, GRS will become a single-user RS when each group has only one member.

The group recommender system has been becoming an increasingly important research area, since the first studies and applications of Masthoff in 2004 [3], [4] on the application of the GRS to television program selection advice introduced, and other GRS applied studies in different areas such as tourism, entertainment services [2], [5]-[8]. GRS will become more and more popular as the need for group decision making for users in collaborative activities becomes more common [9].

Researchs on the group recommender system can be divided into two main approaches: (1) Aggregating individual preferences approach and (2) Aggregating individual recommendations approach. The literature review shows that the second approach is much more dominant than the first approach.

There are many indicators or criteria used to evaluate a GRS [16], such as prediction accuracy, diversity, coverage or consensus and fairness. In the study of group recommender systems, it is shown that when studying GRS the tendency to prioritize the fairness of recommendations is very important.

In addition, to build a GRS reflected closer to reality, it can be seen that research on dynamic group recommender systems using fuzzy computing approach needs to be studied more extensively. Combining the two factors of "dynamic" and "fuzzy computing" can help the GRS problem to correctly represent the characteristics of uncertainty and uncertainty on user evaluations, and the fluctuations in user preference, the changes of product attractiveness over time, thereby helping the model maps and solve real data better in practical.

Based on existing publications on GRS, in fuzzy-based GRS and dynamic GRS certain limitations still persist. Therefore, in this dissertation, the author proposes the development of a ***"Research on developing of Group Recommender System based on approach of intuitionistic fuzzy set and Choquet integral"***. This approach will develop a model of recommendation systems for group user, utilizing fuzzy measures to enhance fairness in recommendations. It will also apply extended fuzzy set theory, Intuitionistic fuzzy set, to better represent and handle the uncertain and ambiguous information in user feedback and evaluations, while considering the dynamic nature of the group recommender system.

2. Research objectives

Research objective: Research on developing dynamic group recommender system using intuitionistic fuzzy set and ensure fairness in recommendation.

3. Main content of dissertation

The main content of the thesis consists three parts presented in three chapters. In which: **Chapter 1** presents the fundementals of the theory of group recommender systems and related issues. On that basis, the thesis analyzes the existing problems and clearly states the research objectives, the proposed methods and the results achieved by the thesis. **Chapter 2** presents the research on group recommender systems with an approach considering fairness based on a fuzzy measure. Combining the two targets of highest total benefits of group members and fairness between members, we will have to solve the multi-objective optimization problem in GRS. **Chapter 3** presents the proposal to use intuitionistic fuzzy set (IFS) theory to develop a dynamic group recommender system. From there, a proposal for a dynamic group recommender system based on intuitionistic fuzzy sets was developed, and in this model of group recommender system, a combination operation with Choquet integral for IFS was further proposed and tested to find a most suitable GRS model for practice.

Chapter 1: OVERVIEW OF GROUP RECOMMENDER SYSTEM

1.1. Introduction on Group recommender system

1.1.1. Group recommender system

The initial Recommender Systems were developed to provide recommendations to individuals, however, nowadays recommender systems are also aimed at providing recommendations to a group of users. Therefore, the application of Group Recommender Systems has been expanding over time [2], [3], [30], [31].

Concept of group recommender system: G can be understood as a recommender system that provides a set of objects (products, services, etc.) that are considered suitable to a group of users [4]. The simplest group recommender system can be modeled as follows.

Given $U = \{u_1, u_2, ..., u_n\}$ and $I = \{i_1, i_2, ..., i_m\}$ are set of users and items; given $\mathrm{R} \subseteq \mathrm{U} \times \mathrm{I} \rightarrow \mathrm{D}$ is set of rating of users given to items on domain D. Let $g = \{u_1, u_2, ..., u_l\} \mid u_i \in U$ is a user group, then the group recommender system is modeled as:

$$HTVN(g, I) = \arg\max_{i \in I}[pref(g, i)] \quad (1.1)$$

where $pref(g, i)$ is predicted preference of group g gives to an item i.

1.1.2. Literature review on Group recommender systems

Group recommender systems can be considered to have started to develop in the late 1990s and early 2000s, with the prominent research of Mathoff et al. [4], and then in recent years, group recommender systems have really become a prominent branch of research.

Early research on GRS focused mainly on developing methods for aggregating individual preferences to generate recommendations for groups [4], [10]. Later, collaborative filtering techniques in GRSs [6], [11], GRS with integrated social influence modeling [12]-[14], and GRS focusing on enhancing diversity and fairness in recommendations [17], [18], [33] were gradually developed.

In addition, improving the way to solve the fairness problem among users in a group will increase the overall user satisfaction, thereby increasing the practical applicability of Group Recommender Systems [21].

There are two common approaches in GRS, the “Aggregating individual preferences” approach and the “Aggregating individual recommendations” approach. The second approach, which is also the more common approach in GRS today [14]. In this approach, the fairness of recommendation generation is controlled during the “consensus” phase of the recommender system.

1.1.3. The “Aggregating individual recommendations” Group recommender system

In the consensus phase, different aggregation operators are used that show strategies in constructing a value that represents the group's preference for an item based on the individual evaluations of a group. The main strategies used in this phase by previous studies include "sum-utility maximization strategies", such as "additive strategies", "average strategies" and "multiplicative strategies"; strategies based on the underdog or the dominant ("least misery strategies" and "most pleasure strategies"); or mechanisms based on actual voting practices such as "Aproval voting strategies" and "Copeland's rule strategies" or a more balanced strategy such as "Borda count strategies" and "fairness strategies" [2], [32].

1.1.4. Group recommender system evaluation

The performance of a GRS can be evaluated through metrics that reflect one of the following aspects: Classification accuracy, prediction accuracy, ranking accuracy, coverage and randomness, consensus and fairness [16]. Among them, consensus and fairness metrics are increasingly considered in Group Recommender Systems.

The concept of fairness in recommender systems in general can refer to fairness between users, fairness between providers, or both [17], [18], [33]. In Group Recommender Systems, studies on fairness tend to focus on the differences in satisfaction levels or ratings among users in a group about recommended items. Several recent studies have proposed definitions and measures for the concept of fairness in GRS, but systematic and in-depth studies in this area are still lacking.

1.2. Literature review on Dynamic, Fuzzy Group recommender system

1.2.1. Dynamic group recommender system

In general, in Recommender Systems, information precessing methods that consider time-effects can be simply divided into four categories [42]. Each category represents a different perspective when processing dynamic and temporal information. The four approaches include: 1) Approximate approach; 2) Clustering-based approach; 3) Online updating method and 4) Dynamic-based approach. Among them, the dynamic-based approach is widely applied. This approach is based on explicit modeling of time-varying variations in feedback to track changing trends of factors such as user preferences and attractiveness of products and services [43, 44].

The review of the research shows that GRS is a later research problem than RS and existing researchs on GRS often focuses on solving the problem of combining member assessments to create group assessments. Research on GRS using dynamic information approach is still relatively limited. Some typical studies can be pointed out such as the research of Jinpeng Chen et al. [52], or Huang's research on the consensus phase of GRS that considers the hierarchical relationship of products over time [53]. It can be seen that research on GRS using dynamic information approach will better reflect the reality of information in the system.

1.2.2. Fuzzy group recommender systems

Utilizing fuzzy theories in building recommender systems is a widely studied strategy. This approach has many advantages such as being able to represent and handle uncertainty in data presents users' evaluation of items [15], [35], [54]. Research on the direct application of fuzzy theories in group recommender systems is somewhat more limited than the application of fuzzy theories in single-user recommender systems.

The literature shows that developing GRS by dynamic approach and developing GRS by fuzzy computing approach both have outstanding advantages, and can support each other. However, the research on these approaches is still quite lacking and needs to be further studied, from which to build a better GRS model.

1.2.3. Introduction about Intuitionistic fuzzy set

1.2.3.1. Overview of intuitionistic fuzzy set

Among the studies on fuzzy sets and extended fuzzy sets, intuitionistic fuzzy set have certain advantages in representing and constructing recommender systems [55]. The definition of intuitionistic fuzzy set was introduced by Atanasov [56], [57].

Definition 1.1: given an universe X, an intuitionistic A on X is as follow:

$$\mathrm{A} = \{(\mathrm{x}, \mu_A(\mathrm{x}), v_A(\mathrm{x})) \mid \mathrm{x} \in \mathrm{X}\} \quad (1.17)$$

where: $\mu_A : X \to [0,1], v_A : X \to [0,1]$. In which, $\mu_A(x) \in [0,1]$ presents degree of membership of x, and $v_A(x) \in [0,1]$ presents degree of non-membership x, and the constraint $0 \leq \mu_A(x) + v_A(x,y) \leq 1$ hold with $\forall x \in X$.

The algebraic operations for intuitionistic fuzzy sets have been introduced in [58]. These algebraic operations in intuitionistic fuzzy sets are the foundation for developing algorithms for processing intuitionistic open data.

1.2.3.2. Distance and similarity of intuitionistic fuzzy sets

Given A, B are two intuitionistic fuzzy sets on $X = \{x_1, ..., x_n\}$

Hamming distance:

$$d_{IFS}(A,B) = \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \quad (1.20)$$

Euclidean distance:

$$e_{IFS}(A,B) = \sqrt[2]{\sum_{i=1}^{n} \left((\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2 \right)} \quad (1.21)$$

1.2.3.3. Intuitionistic fuzzy set Mean

Below are important aggregation and mean operations..

Given $A=\left\{\left(\mu_{a_i},\nu_{a_i}\right);i=1,2,...,n\right\}$ is set of intuitionistic fuzzy numbers. We have:

Intuitionistic arithmetic weighted Mean:

$$IFAW(A,W)=\overset{n}{\underset{i=1}{\oplus}}a_i w_i=\left\langle 1-\prod_{i=1}^{n}\left(1-\mu_{a_i}\right)^{w_i},\prod_{i=1}^{n}\nu_{a_i}{}^{w_i}\right\rangle \tag{1.30}$$

Intuitionistic Bonfferroni Mean [62]

$$IFB^{p,q}(A)=\left(\frac{1}{n(n-1)}\left(\overset{n}{\underset{\substack{i,j=1\\ i\neq j}}{\oplus}}a_i^p\otimes a_j^q\right)\right)^{\frac{1}{p+q}} \tag{1.31}$$

1.2.4. Choquet integral

To construct a matching operation for the consensus phase of a group recommender system based on the Choquet integral, we need to present again how to calculate the Choquet integral.

a) Choquet integral :

Definition 1.2: Choquet integral of a vector $r\in\mathbb{R}^m$ using a capacity funciont ξ is defined as follow:

$$CQ_\xi(r)=\sum_{i=1}^{m}\left(r_i^{\uparrow}-r_{i-1}^{\uparrow}\right)\xi(\Upsilon_i^{\uparrow}) \tag{1.33}$$

where $r^{\uparrow}=\left\{r_1^{\uparrow},...,r_m^{\uparrow}\right\}$ is a permutation in ascending order of r, in which $0=r_0^{\uparrow}\le r_1^{\uparrow}\le...\le r_m^{\uparrow}$, and the set $\Upsilon_i^{\uparrow}=\left\{j\in \mathrm{M},r_j\ge r_i^{\uparrow}\right\}=\left\{i^{\uparrow},(i+1)^{\uparrow},...,m^{\uparrow}\right\}$ with $i<m$ and $\Upsilon_{m+1}^{\uparrow}=0$.

b) Fuzzy measure:

When constructing a aggregation based on the Choquet integral, if the capacity function is non-additive, the aggregation represents a fuzzy measure that reflects the goal of the aggregation [41,64]. Constructing an optimal fuzzy measure is an NP-complete problem and is therefore not feasible to solve with complete algorithms.

In a Group Recommender System, to apply the Choquet integral-based fusion operation, it requires computing the capacity function value for each group of users in a reasonable amount of time. A feasible approach is to propose an algorithm that directly evaluates the value of the points in the required linear extension and it should be a computationally efficient process. This thesis follows this approach and determines some capacity functions based on user interactions and the goal of increasing the fairness in GRS recommendations.

1.3. Summary of chapter 1

Chapter 1 presents some fundamental of group recommender systems based on the generalization of single-user recommender systems. In the overview of research on group recommender systems, including approach strategies, evaluation methods, studies using static information, dynamic information and studies using fuzzy computing approaches are presented. The approaches are presented and analyzed according to the advantages and disadvantages of each method. On that basis, chapter 1 presents the research problems of the thesis. Specifically, the thesis focuses on group recommender systems and proposes and develops a group recommender system algorithm using fuzzy measures based on Choquet integrals to improve the fairness of recommendations in chapter 2 and proposes and develops a dynamic fuzzy approach in chapter 3.

Chapter 2 INCREMENTAL OF GROUP RECOMMENDER SYSTEM BY USING FUZZY MEASURES

2.1. Introduction

With HTVN, the issue of fairness in recommendations is a matter of particular concern [16], [18], [66]. These studies have proposed a number of proposals, including considering the fairness of HTVN as the ratio of satisfied people to the total number of group members [10], the deviation of the satisfaction level of group members [19], or considering the fairness of the recommended product set as a "package" rather than a set of independent products [18].

In addition, another challenge posed in finding a good fairness solution in a consensus-based GRS is that a member's preference for a product or service is influenced by the interaction of members [3], [67], [68]. Therefore, to estimate the imbalance between the preferences of group members, it is necessary to take into account the interaction of members.

In the consensus phase, instead of the previous union operations, the thesis proposes to use Choquet integral to generate group proposals in the consensus phase of GRS. The aggregation operation based on Choquet integral expands the solution search scope compared to weighted aggregation and it can give more balanced recommendations than previous strategies by constructing a suitable fuzzy measure [41], [70].

2.2. Proposed GRS model using aggregation operator based on Choquet integral

2.2.1. GRS models with aggregation operator based on Choquet integral

a) Proposed GRS model based on Choquet integral

In this thesis, the researcher proposes a two-phase HTVN model, the first phase is the recommendation generation phase that predicts a user's rating of products, and the second phase is the phase that represents the consensus mechanism among members in a user group. Specifically:

- Recommendation phase uses user-based collaborative filtering.

- Consensus phase: uses the Choquet integral-based aggregation operator to estimate the group's rating of products and services based on the rating of each member. Based on the group's rating results, recommendations will be made to the group according to the principle of selecting the highest rated product.

b) Proposed capacity functions

First proposed capacity function:

Below, the authors propose a capacity function called "first-order capacity function" based on the level of user interaction with the system. This study is based on the proposal in the study of Huynh et al. [78]. Let the user group be considered as a condition when selecting products. The capacity function is defined as follows:

$$\xi(u_i) = \begin{cases} \dfrac{\omega(u_i)}{\sum\limits_{u_i \in g} \omega(u_i)}, \text{if } \sum\limits_{u_i \in g} \omega(u_i) < 1 \\ \omega(u_i), \text{ otherwise} \end{cases} \tag{2.3}$$

where:

$$\omega(u_i) = \frac{count(r_{u_i j} \geq 0)}{|I|}, j = 1,...,|I| \tag{2.4}$$

And the capacity function value of a sub set $A \subset g$ is:

$$\xi(A) = \sum_{u_i \in A} \xi(u_i) + \sigma(A) \tag{2.5}$$

And if $\xi(A) > 1$ the it is set to 1.

Lema 2.1: The capacity function defined above satisfies the additive property when $\sigma(A) = 0$ and $\sum_{u_j \in g} \omega(u_j) \leq 1$.

Properties 2.1: In case the capacity function defined above is additive, the Choquet integral-based aggregation operator becomes a weighted sum aggregation.

In common of the first capacity function we have $\sigma(A) \neq 0$, and it is calculated by the following formula.

$$\sigma(A) = \sum_{u_i \in A} \omega'(u_i) \tag{2.6}$$

where:

$$\omega'(u_i) = \frac{count(r_{u_i j} > aveRate) - count(r_{u_i j} < aveRate)}{|I|} \tag{2.7}$$

The item set I includes all items rated by user u_i. $\omega'(u_i)$ represents a user's level of interest in evaluating products.

Second proposed capacity function:

In group activities, interactions between small groups of users will affect a person's satisfaction. For the same activity, a person may feel more satisfied when participating with others who are highly similar to him/her. Therefore, NCS proposes an expanded capacity function as follows.

$$\sigma'(A) = \frac{1}{dens(A)} * \sigma(A) \tag{2.8}$$

Density represents the similarity between a group of users and it is based on the distance of all users, and is calculated based on the Mahalanobis distance measure.

2.2.2. Time complexity of proposed model

To compare the complexity of the new proposed algorithm for the consensus phase using differen consensus strategies presented in section 1.2.2, we need to evaluate the complexity of the algorithm based on the concept of big-O notation that present worst case complexity of an algorithm. Through the complexity evaluation from the pseudocode of the algorithm, we can conclude that the complexity of the consensus phase using the Choquet integral union operator will be $O\left(|I|^2 .|g|.N\right)$.

2.3. Experiment and discussion

2.3.1. Data set

To compare the proposed approach and other approaches, MovieLens-1M data is used. Therefore, when applying in the GRS problem, it is necessary to build a user clustering mechanism. In this study, with the clustering, a random sampling mechanism is used to select users $|g_i|$ to create hypothetical user groups.

2.3.2. Measurement matrics

Group satisfaction:

Note that one goal of HTVN is still to find products with high total satisfaction in addition to fairness, so we need a measure that represents the satisfaction value of the entire group of users. The formula for calculating this measure is as follows:

$$group\_pref\left(g,I\right)=\frac{\sum_{i\subset I} r_{g,i}}{|I|} \tag{2.13}$$

Where $r_{g,i}$ present the average satisfaction of all members of group g to an item $i \in I$.

Fairness measure: To estimate the fairness of recommendations from the GRS model, the thesis uses two measures shown in formula (1.14) as follows.

$$fairness(g,i) = \frac{\left| \bigcup_{u \in g} : r_{ui} \geq \theta \right|}{|g|}$$

In which θ is a threshold that presents the user has rated an item high than this threshold is an user that satify with a recommende item.

And the second fairness measure present in formula **Error! Reference source not found.** as follows:

$$fairness_{Var}(g,i) = 1 - Var\{r_{ui}, \forall u \in g\}$$

2.3.3. Experiment and discussion

Some of the key results are shown below.

Table 0.4 Average rating of group

Top-N	AUS	MS	LMS	MPS	AVS	CRS	CIS_CF1	CIS_CF2
N=1	**4.717**	4.694	4.373	4.384	4.412	4.717	4.556	4.560
N=2	**4.689**	4.670	4.350	4.404	4.383	4.689	4.536	4.537
N=3	**4.669**	4.658	4.339	4.387	4.357	4.669	4.514	4.516
N=4	**4.652**	4.645	4.337	4.396	4.350	4.652	4.503	4.505
N=5	**4.637**	4.628	4.331	4.397	4.349	4.637	4.481	4.483
N=6	**4.625**	4.615	4.327	4.401	4.342	4.625	4.465	4.467
N=7	**4.614**	4.605	4.313	4.402	4.341	4.614	4.451	4.453
N=8	**4.600**	4.594	4.301	4.398	4.339	4.600	4.442	4.444
N=9	**4.586**	4.578	4.292	4.390	4.335	4.586	4.433	4.433
N=10	**4.572**	4.564	4.279	4.378	4.331	4.572	4.420	4.420

Table 2.4 shows the difference between the models in terms of the average rating of the users in the group. Obviously, the AUS strategy has the best result when considering the average satisfaction level of all the users in the group. GRS with the Copeland strategy has the same performance as AUS. The two proposed algorithms have results very close to AUS and they are better than the other strategies.

Figure 2.2 shows the difference in average group ratings according to recommendation size showing that the difference of algorithms decreases as the recommendation size increases.

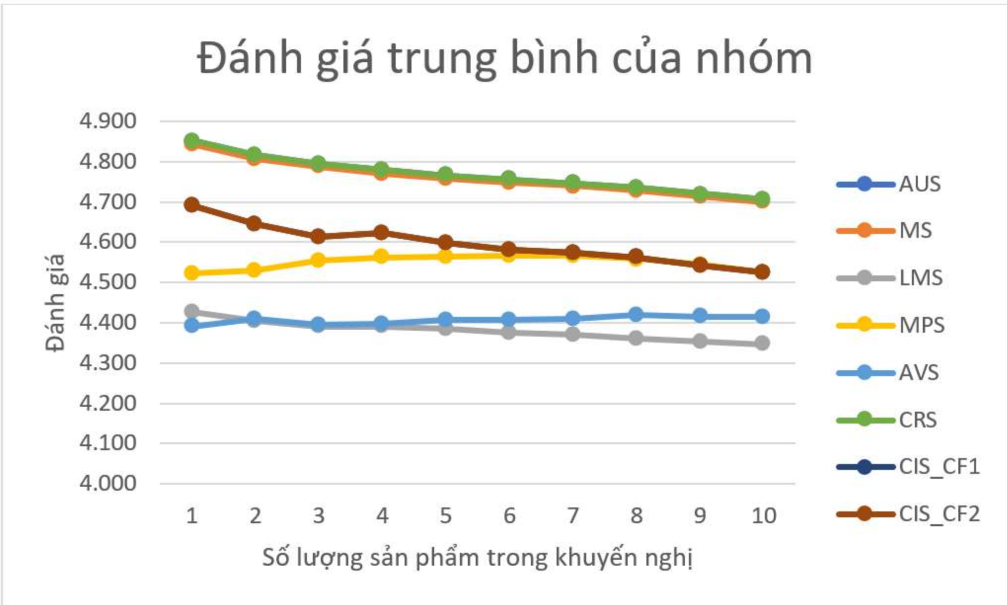


Figure 0.2 Average group satisfaction

In terms of fairness, we have the fairness of consensus strategies shown in the following figure:

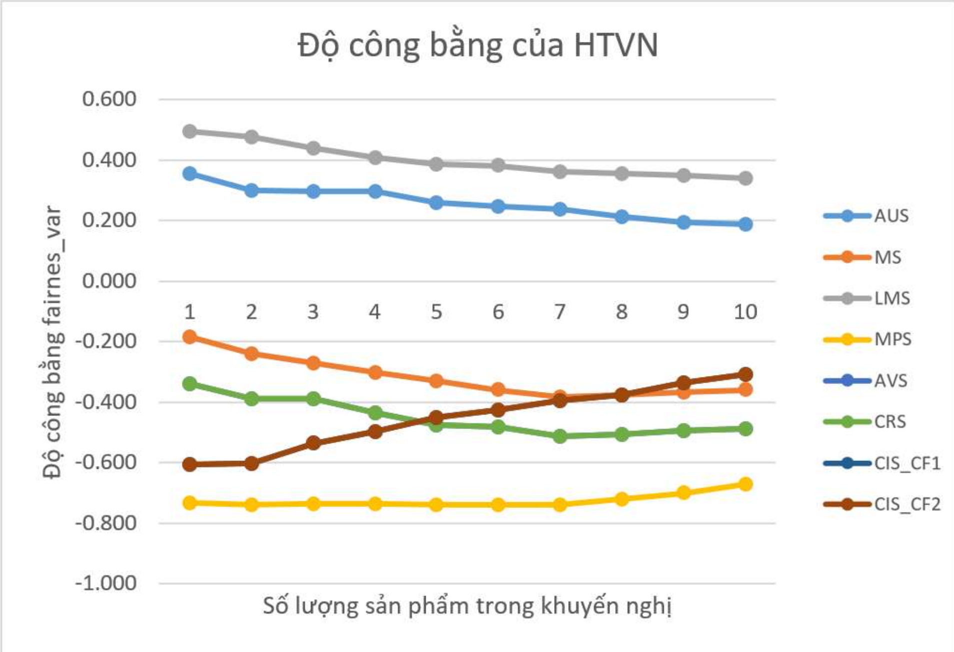


Figure 0.5 Fairnes of GRSs

We can see that the "least disadvantage strategy" (LMS) and the "approval voting strategy" (AVS) have the highest fairness factor, but they in contrast choose the items with the lowest average user satisfaction. The "dominant strategy" (MPS) gives the least fairness solution in all cases and it also gives recommendations with very low average group preferences. The "additive strategy" (AUS) and the Copeland's rule strategy (CRS) give recommendations with the highest average group preferences but they also give recommendations with low fairness. GRS with the two proposed Choquet integral-based fusion operations (CIS_CF1 and CIS_CF2 – corresponding to two different capacity functions) does not outperform the other models in any single aspect of the two aspects: average group preferences and fairness measure. However, it can be easily seen that these two proposed models balance both of the above objectives. Therefore, the proposed GRS model will have high applicability in practice.

2.4. Summary of chapter 2

Chapter 2 studies the construction and development of a group recommender system based on the Choquet operator to increase the fairness of recommendations. The proposed model is based on the approach of building a group recommender system with a consensus phase. In the consensus phase, the Choquet operator is used to develop a union operation of the individual ratings of users in the group. With the application of the Choquet operator, the consensus phase in GRS has solved the problem of interaction between group members, and represented a fuzzy measure expressed in the union operation. From there, the recommendations generated by GRS will have a more balanced satisfaction among all users in the group.

Chapter 3 DYNAMIC GROUP RECOMMENDER SYSTEM USING INTUITIONISTIC FUZZY SET AND ENSURE FAIRNESS

3.1. Introduction

In this thesis, intuitionistic fuzzy sets are the focus of research to build a suitable representation for information about user evaluation of products and services [89]–[91]. In addition, information about user evaluation is considered under dynamic information approach. These are two important factors to be able to build GRS closer to reality.

In this chapter three, the researcher presents a study on "Dynamic Group Recommender System based on Intuitionistic Fuzzy set and ensure Fairness". The main objectives to be achieved are as follows:

- Research on Dynamic Group Recommender System to develop a method to handle changes in user preferences and the decline in product attractiveness.

- Research on Dynamic Group Recommender System based on Intuitionistic Fuzzy Set to handle information about hesitation and uncertainty in HTVN.
- Research and propose a dynamic HTVN model on intuitionistic fuzzy set in which the union operation uses Choquet integral as the basis to ensure the fairness of the recommendation results.

3.2. Proposed Dynamic GRS based on Intuitionistic Fuzzy Set

3.2.1. Proposed Dynamic GRS based on IFS:

a. General model

Based on the approach of building a group recommender system with a consensus phase used to generate a common recommendation for a group of users, the dynamic group recommender system model on intuitionistic fuzzy set is shown in the following diagram:

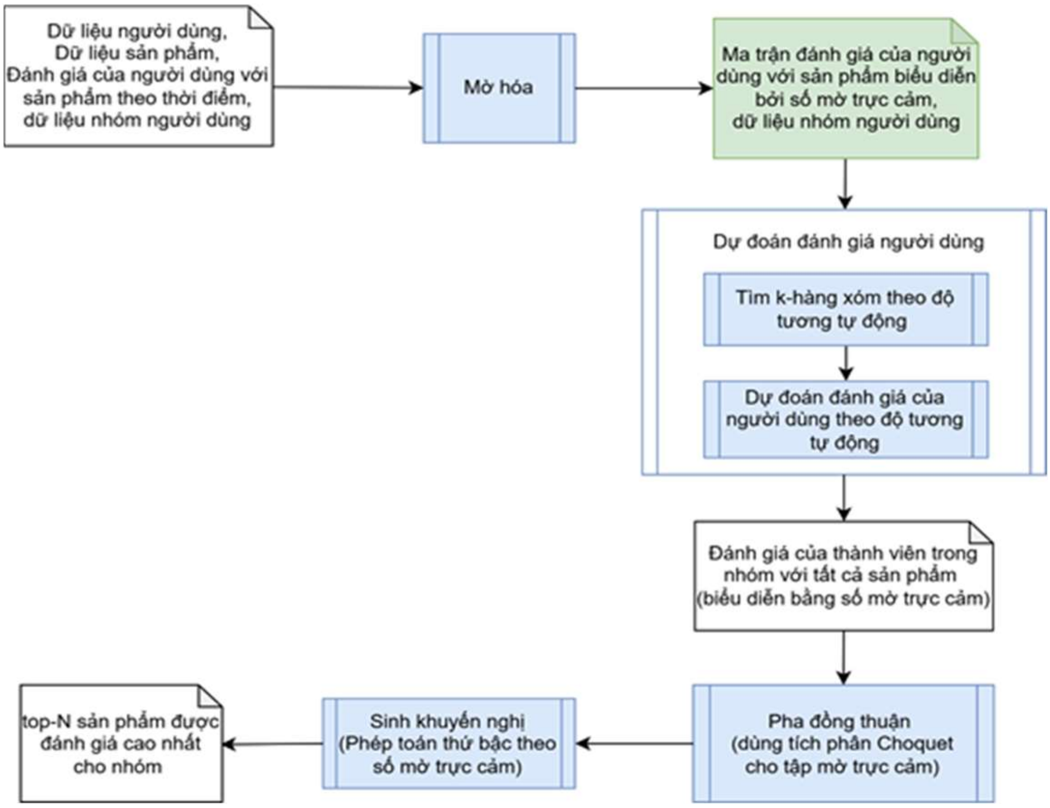


Figure 3.1 GRS process

b. Detail of operation in the proposed model

Step 1: Fuzzilize the user rating matrix

Step 2: Calculate dynamic similarity

Definition 3.1: given t_{ui}, t_{uj} are time points that users u_i, u_j rates an item k. The time effect on users' similarity is estimated by the following formula:

$$f(\Delta t_i) = \frac{1}{1+\lambda\Delta t} = \frac{1}{1+\lambda\left|t_{ui} - t_{vi}\right|} \tag{3.4}$$

And the dynamic similarity is shown as follows:

$$Dsim(u,v) = Sim(u,v).f(\Delta t_i) \tag{3.5}$$

Bước 3: Predict user ratings considering the influence of time :

In order to estimate the rating a user u gives to an item j (presented by an intuitionistic fuzzy number) we can use one in two following formulas.

$$a_{ui} = \bigoplus_{j=1}^{n} \tfrac{1}{n} a_{uj} \oplus IFAW(\tilde{A}_v, W_{DCossim(v,u)}) \tag{3.7}$$

$$a_{ui} = IFB^{p,q} a_u \oplus IFAW(\tilde{A}_v, W_{DC\,os\,sim(v,u)}) \tag{3.8}$$

Where:

$$IFAW(\tilde{A}_v, W_{DC\,os\,sim(v,u)}) = \bigoplus_{v \subset \hat{U}} \tilde{a}_{vi} D\cos sim(v,u) \tag{3.9}$$

3.2.2. Consensus phase of proposed Dynamic GRS based on Intuitionistic fuzzy set

Choquet integral on intuitionistic fuzzy set:

Definition 3.3: Given a group users $g = \{u_1, u_2, ..., u_n\}$, and a vector $R_g = \left\{\left(\mu_{u_i}, \nu_{u_i}\right); i = 1, 2, ..., n\right\}$ is a set of intuitionistic fuzzy numbers that represent the judgments of members of a group g about an item. Let ξ is a capacity function on g, we have an aggregation operation of GRS based on Choquet integral as follows:

$$\text{IF\_CA}_{\xi}(R_g) = \left(1 - \prod_{i=1}^{n}(1-\mu_{u_i})^{\xi(A_i^{\uparrow})-\xi(A_{i+1}^{\uparrow})}, \prod_{i=1}^{n}\nu_{u_i}^{\ \xi(A_i^{\uparrow})-\xi(A_{i+1}^{\uparrow})}\right) \quad (3.13)$$

Where $A_i^{\uparrow}$ is sub-set of group members, particularily $A_i^{\uparrow} = \{u_i, u_{i+1}, ..., u_n\}$ is subset of g in ascending order of rattings, $A_{n+1}^{\uparrow} = \phi$.

3.2.3. Parameter learning method:

In the DIFGRS algorithm, some parameters will affect the effectiveness of predicting a user's rating of products and services. In this study, Bayesian optimization algorithm is used to learn the important parameters of the system.

3.3. Experiment

3.3.1. Experimental data

Experimental data of the dynamic fuzzy recommender algorithms are performed on the same MovieLens-1M dataset presented in chapter 2.

3.3.2. Measurement metrics

Average group satisfaction:

The members' evaluations are all represented by intuitionistic fuzzy numbers, so the value representing the group's average satisfaction with all the products I in the recommendation is calculated by the following formula.

$$g\_pref(g, I) = \frac{1}{|I|}\sum_{i \subset I}\left(\frac{\sum_{u \in g} utility\_score(r_{ui})}{|g|}\right) \quad (3.27)$$

Measures of fairness

- **Ration of satisfied users:**

The first fairness measure used is the ratio of members satisfied with the recommendation to the total number of people in the group.

$$fairness(g,i)=\frac{\left|\bigcup_{u\in g} r_{ui}\succ r^{th}{}_{i}\right|}{|g|} \tag{3.28}$$

where $r^{th}{}_{i}$ is the average rating of each item calculated using the following formula.

$$r^{th}{}_{i}=IFAW(R_i,W)=\overset{n}{\underset{u=1}{\oplus}} r_{ui}w_u=\left\langle 1-\prod_{u=1}^{n}\left(1-\mu_{r_{ui}}\right)^{w_u},\prod_{u=1}^{n}\nu_{r_{ui}}{}^{w_u}\right\rangle \tag{3.29}$$

- **Fairnese measure:**

The second measure is used to assess the difference in satisfaction levels of individual group members. This measure is called the fairness measure, and is calculated as follows.

$$equity\left(g,I\right)=\frac{1}{|I|}\sum_{i\subset I}\left\{\frac{1}{|g|^2}\sum_{l=1}^{|g|}\sum_{k=1}^{|g|}dis(r_{li},r_{ki})\right\} \tag{3.30}$$

- **Equity measure:**

The third fairness measure is a newly proposed measure based on the concept of GINI index which represents the degree of inequality among a group of users. In the dynamic fuzzy group recommender system, the inequality measure is proposed and calculated as follows:

$$Gini\_equity\left(g,I\right)=\frac{\sum_{l=1}^{|g|}\sum_{k=1}^{|g|}\left|utility\_score(r_{li})-utility\_score(r_{ki})\right|}{2|g|\sum_{u=1}^{|g|}utility\_score(r_{ui})} \tag{3.31}$$

3.3.3. Result and discussion

In this part of the thesis, the efficiency of the proposed consensus algorithm using Choquet-based matching operation for dynamic fuzzy GRS (abbreviated as IF_CIS) will be presented and compared with other consensus strategies for HTVN on intuitionistic fuzzy sets.

The comparison results include analysis of the average rating of the user group with the recommendation, the fairness of the recommendation according to different fairness measures. Below is a summary of some of the main comparison results.

On average satisfaction level by group:

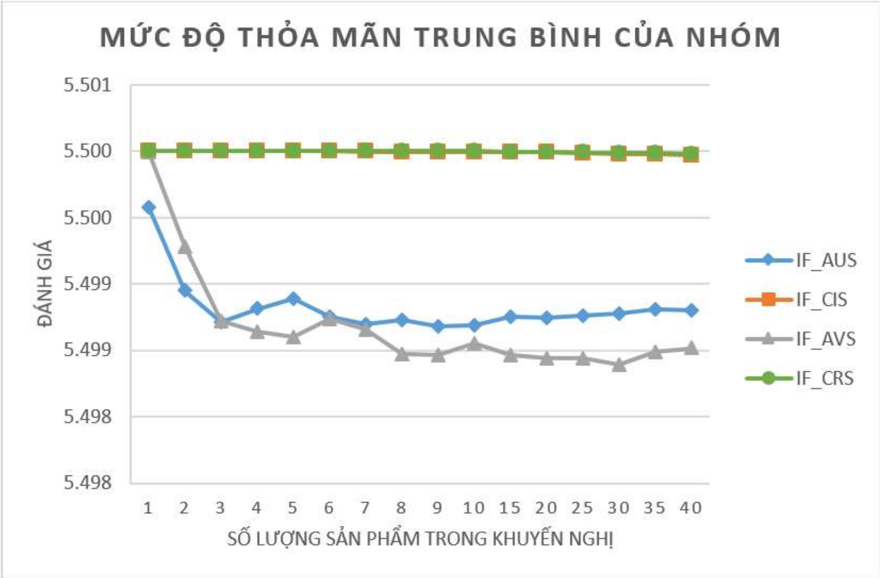


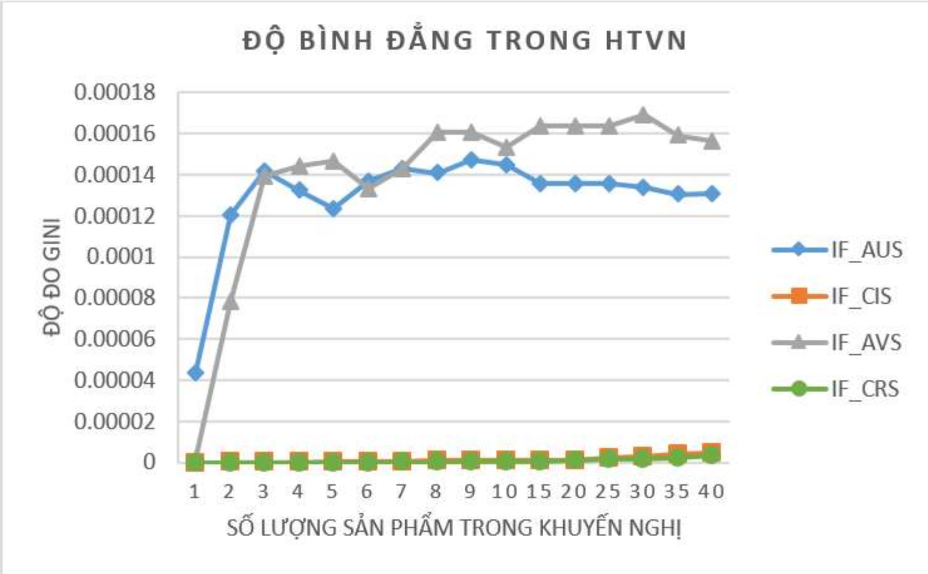
***Figure** 0.4* Average group satisfaction by four best GRSs

Figure 3.4 shows a small difference between the four best GRSs, where the experiments show that the dynamic fuzzy GRSs using the consensus phase IF_CIS and IF_CRS outperform the other two approaches. This shows that the algorithm using the Choquet integral is as effective as the best algorithm in terms of group average satisfaction.

On the fairness of group recommendations

Figure 3.6 above shows the performance of four algorithms that are superior in terms of the second fairness measure. Among these four algorithms, IF_AUS, IF_CIS, IF_AVS and IF_CRS, it is easy to see that the two algorithms IF_CIS and IF_CRS give better results. From this, we can see that the dynamic fuzzy GRS algorithm using the union operator with Choquet integration (IF_CIS) gives the best results.

Figure 0.6 Fairness of the four best dynamic fuzzy GRSs



In terms of GINI equity:

Figure 0.8 GINI fairness of the four best dynamic fuzzy GRSs

Figure 3.8 above shows the performance of four algorithms that are superior in terms of the third fairness measure: inequality. Among these four algorithms IF_AUS, IF_CIS, IF_AVS and IF_CRS, it is easy to see that the two algorithms IF_CIS and IF_CRS give better results. From this we can see that the dynamic fuzzy GRS algorithm using the union operator with Choquet integration (IF_CIS) gives the best results in terms of the third fairness measure.

3.4. Summary of chapter 3

Research on the construction and development of a dynamic group recommender system based on intuitionistic fuzzy sets. Dynamic information is processed according to the assumption that user preferences and product attractiveness are time-varying information. In addition, in the forward dynamic phase of the model, the Choquet operator is still used to develop a mathematical operation that combines the individual ratings of users in the group.

NEW CONTRIBUTIONS OF THE THESIS

The thesis studies the Group Recommender System, a generalized recommender system of the traditional single-user recommender system, following the intuitionistic fuzzy set approach. In which the challenges of the group recommender system such as the fairness of recommendations, dynamic information, hesitation and uncertainty of information are studied and proposed solutions. Based on theoretical and experimental research, the main results achieved in the thesis include:

1. Propose an algorithm for the consensus phase using Choquet integrals to build a group recommender system to enhance the fairness of recommendations and ensure the overall user benefit in the group is maintained.

2. Propose a group recommender system based on a dynamic model using Choquet integrals and intuitionistic fuzzy set in the consensus phase to respond to solving problems where user ratings change over time.

The results of the thesis can be applied in practice to develop intelligent information systems, providing the ability to filter appropriate information and make recommendations for user groups.

LIST OF THE PUBLICATIONS RELATED TO THE DISSERTATION

1. **Cu Nguyen Giap**, Nguyen Nhu Son*, Nguyen Long Giang, Hoang Thi Minh Chau, Tran Manh Tuan, Le Hoang Son, "A New Approach for Fairness Increment of Consensus-driven Group Recommender Systems Based on Choquet Integral", International Journal of Data Warehousing and Mining (IJDWM), 2022, 18(1), 1-22. (ISSN 1548-3924, **SCIE, IF: 1,2**).

2. **Cu Nguyen Giap**, Le Thi Huyen Dieu, Luong Thi Hong Lan, Tran Thi Ngan, Tran Manh Tuan, "Utilize Deep learning to increase the performance of a Book recommender system using the Item-based Collaborative Filtering". Hội thảo quốc tế RICE, 2022, 109-113.

3. Nguyễn Như Sơn, **Cù Nguyên Giáp**, Lê Hoàng Sơn , Nguyễn Long Giang, Dương Thị Thanh Loan, Trần Mạnh Tuấn, Dương Thị Thu Huyền, "Kỹ thuật tư vấn nhóm dựa trên tập mờ trực cảm và ưng dụng", Hội thảo Quốc gia lần thứ XXV "Một số vấn đề chọn lọc của Công nghệ thông tin và Truyền thông" (2022), Hà Nội, Việt Nam, 2022, 81-86.

4. Nguyen Nhu Son, **Cu Nguyen Giap***, Le Hoang Son*, Nguyen Long Giang, Tran Manh Tuan, Vassilis C. Gerogiannis, Dimitrios Tzimos, "A Dynamic Fuzzy Group Recommender System based on Intuitionistic Fuzzy Choquet Integral Aggregation", Soft Computing, 2024, 1-14. (E-ISSN 1433-7479, **SCIE, IF: 4,1**).