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THE DISCRETIZED KALUZA-KLEIN THEORY AND DARK DECAY CHANNELS OF NEUTRON

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INTRODUCTION

1. Motivations of the thesis

Standard model (SM) has very successfully described the strong, weak and electromagnetic interactions with predictions consistent with the experiments. There are three main trends expanding SM to unify interactions:

- *Expanding the gauge symmetry groups* $SU(3)_c \times SU(2)_L \times U(1)_Y$.
- *Expanding the 4-dimensional space-time with extra dimensions*.
- *Expanding the effective theory of gravity*.

Cartan developed a formalism based on Riemannian geometry that replaced Einstein's equations with two Cartan equations.

Non-commutative geometry (NCG) was postulated by Connes-Lott as an extension of Riemannian geometry. The SM in this space-time dynamically includes a Higgs scalar field with a fourth-order potential energy, causing a spontaneous gauge symmetry violation mechanism. This model contains fewer free parameters than the conventional SM, thus predicting the Weinberg angle and top quark mass.

The extended theory of gravity in Connes-Lott’s space-time includes gravitational and electromagnetic interactions, similar to the traditional Kaluza-Klein theory. Therefore, Viet-Wali proposed a theory called “Discretized Kaluza-Klein Theory” (DKKT) [1]. It is based on the structures of NCG and the Kaluza-Klein (KK) theory with the 5^{th}-dimension consisting of only two discontinuities to build a gravitational theory using the Cartan formalism. Thus, DKKT has overcome the disadvantage of the KK theory that it only contains a limited number of the KK partner fields. In DKKT, all known interactions are part of gravitational interactions in the extended space-time with the discrete extra dimensions.

Another direction that needs to be prioritized for development is to come up with a theory for dark matter particles that can be observed on the Pelletron accelerator in the Department of Physics, University of Natural Sciences. The scientists at the Center for Nuclear Physics conducted independent verification measurements based on the discovery of the X17 anomaly at ATOMKI in 2019 and obtained positive results. [2].

With the desire to apply DKKT to unify interactions and explain experimental problems, I decided to choose the topic **"Discretized Kaluza-Klein theory and dark decay channels of neutron"** for research.

2. Research purposes

The DKKT includes both gravitational and electromagnetic interactions but does not include strong and weak interactions. Therefore, it is necessary to include the non-Abelian gauge fields in the model. This thesis focuses on examining the cases of non-Abelian gauge fields on two sheets corresponding to right-handed and left-handed particles..

Next goal, we will consider the neutron decay problem in the new elementary particle model. From there, it is possible to simultaneously explain the neutron lifetime puzzle and establish conditions for the model parameters

3. Object and scope of the research

Extending DKKT to the case that includes the non-Abelian gauge fields. The non-Abelian Yang-Mills fields with the goal of describing the strong and electroweak interactions appear as components of the metric in the extended space-time. Building the theory for dark matter such as the X17 boson vector discovered at ATOMKI to solve the neutron decay problem by considering new dark decay channels.

4. Research content

We consider the general form of the geometric quantities such as the connection, torsion and curvature of the extended gravitational theory in the

Cartan representation. Apply DKKT in case the components of the metric are non-Abelian gauge fields.

Based on the extended elementary particle model with Kaluza-Klein companions in DKKT theory, we consider decay channels other with β-channel of neutron.

5. Research methods

The DKKT theory is based on the NCG and the Cartan formalism of the general relativity.

Quantum field theory and Feymann diagram rules to calculate the decay width of neutron decay channels in the model.

Use mathematical tools such as Mathematica software to calculate numerically.

6. Structure of the thesis

Excluding the introduction, conclusions and referrences, the main content of the thesis get 4 chapters.

Chapter 1. Research overview: Brief introduction to the scientific and practical basis. Brief introduction to the two foundations of DKKT theory: Cartan formalism and NCG.

Chapter 2. Discretized Kaluza-Klein theory DKKT: General presentation of DKKT and its new elementary particle model.

Chapter 3. Non-Abelian gauge fields as components of gravity in DKKT: Non-Abelian gauge fields appear as components on the two sheets of the general vierbien. Strong and weak interactions can combine with gravity in the Hilbert-Einstein action.

Chapter 4. Decay width of neutron in the new decay channels of the DKKT: Explaining the neutron lifetime puzzle. Comparison of the contributions of the new decay channels.

CHAPTER 1. RESEARCH OVERVIEW

1.1. The scientific and practical basis

1.1.1. Foundations of modern physics and Unified Field Theory

Modern physics is based on two pillars: Relativity Theory and Quantum Theory. The unification of Quantum Mechanics with Special Relativity Theory into Relativistic Mechanics and Quantum Field Theory took place relatively smoothly.

Extending Quantum Field Theory to fit GR has encountered the insurmountable difficulties including: quantizing the gravitational field and building a common mathematical structure for SM and GR.

1.1.2. Standard Model

SM [3, 4, 5] by Salam, Weinberg and Glashow constructed for three generations of lepton and quarks, each generation consisting of quark-leptons $(q_{Lc}, u_{Rc}, d_{Rc}, l_L, e_R)_A$, generation indexes A=1,2,3 and color indexes c = r, y, b. Bilinears $SU(2)_L$ quark $q_{LA} = (u_L, d_L)_A$ and lepton $l_{LA} = (e_L, \nu_L)_A$ helicity left-handed. The arrangement of left- and right-handed quarks and leptons is based on the violation of parity of the weak interaction [6, 7].

In SM, the interaction between quarks and leptons is built from the gauge fields of the symmetry group $SU(3)_c$ x $SU(2)_L$ x $U(1)_Y$. The theory for these gauge fields is a combination of two independent gauge theories: the Yang-Mills theory unifying the weak interaction and the electromagnetic interaction; QCD describes the strong interactions between quarks.

1.1.3. Multi-dimensional space theory

The Kaluza-Klein theory is a general relativity theory extended in 4+1 dimensional space. The result of the theory is that the electromagnetic and gravitational interactions are unified into the metric of 5-dimensional space [8] and appear infinitely massive partners of the known particles [9].

1.1.4. Dark matter

"Bright" matter including quarks and leptons of the SM accounts for less than 5% of the matter in the universe, while dark matter accounts for 25%, and the remaining is dark energy [10]. Therefore, we need to extend SM to obtain a theory that includes dark matter. The Kaluza-Klein partners could be considered a potential candidate.

1.2. Mathematical basis

1.2.1. Riemannian geometry and Cartan formalism

Rieman geometry is based on manifolds and Cartan formalism is based on vierbein [11].

Algebra of smooth functions on 4-dimensional Riemannian manifolds $\mathcal{M}^4$ is defined as follows:

- 0-forms are scalar functions that are differentiable infinite times
$$f(x): M^4 \to R,\ f(x) \in C^\infty(M^4). \tag{1.1}$$
- The external derivative d is defined on the zero forms as follows
$$df = dx^\mu \partial_\mu f(x), \tag{1.2}$$
- The 1-forms module is an extension of the df envelope that includes forms
$$A = dx^\mu A_\mu(x). \tag{1.3}$$
- The wedge product of two 1-forms is defined as a 2-form below
$$A \wedge B = dx^\mu \wedge dx^\nu\ \tfrac{1}{2}\left(A_\mu(x) B_\nu(x) - A_\nu(x) B_\mu(x)\right). \tag{1.4}$$
$$dx^\mu \wedge dx^\nu = -dx^\nu \wedge dx^\mu. \tag{1.5}$$
- In particular, we can define the action of the external derivative d on

1-forms as follows:

$$dA = dx^{\mu} \wedge dx^{\nu} F_{\mu\nu}(x), \quad F_{\mu\nu}(x) = \tfrac{1}{2}\left(\partial_{\mu} A_{\nu}(x) - \partial_{\nu} A_{\mu}(x)\right). \tag{1.6}$$

That means the external derivative satisfies the de Rham condition $d^2 = 0$

- Vierbein are the transformation coefficients in the local Lorentz rotation transforming the curved coordinate system $x_{\mu}, (\mu = 0,1,2,3)$ into a local flat coordinate system $x_a, \left(a = \dot{0},\dot{1},\dot{2},\dot{3}\right)$

$$x_a = e_a^{\mu}(x) x_{\mu}, \quad x_{\mu} = e_{\mu}^{a}(x) x_a, \tag{1.7}$$

where $e_{\mu}^{a}(x)$ is the inverse of $e_{a}^{\mu}(x)$

$$e_b^{\mu}(x) e_{\mu}^{a} = \delta_b^a \tag{1.8}$$

- The common metric is expressed in terms of vierbein as follows

$$g_{\mu\nu}(x) = e_{\mu}^{a}(x) \eta_{ab} e_{\nu}^{a}(x), \quad \eta_{ab} = diag(-1, 1, 1, 1). \tag{1.9}$$

Thus vierbein can be expressed in the 1-form as follows

$$e^a = dx^{\mu} e_{\mu}^{a}(x). \tag{1.10}$$

The geometric quantities are connection, Ricci curvature and torsion which are determined by the following equations:

- The first Cartan equation

$$T^a = de^a - e^b \wedge \omega_b^a, \tag{1.11}$$

with 1-forms $\omega_{ab} = \eta_{bc} \omega_a^{\ c} = dx^{\mu}\, \omega_{ab\mu}(x)$ are connections that satisfy the condition

$$\omega_{ab} = -\omega_{ba}. \tag{1.12}$$

- The second Cartan equation

$$R_{ab} = d\omega_{ab} + \omega_{ac} \wedge \omega_b^c, \tag{1.13}$$

is used to calculate the components of the curvature Ricci 2-form according to the components of the connection.

- Ricci scalar curvature is given by

$$R = \eta^{ac} R_{ab\mu\nu}(x) e_c^{\mu}(x) e_d^{\nu}(x) \eta^{bd}, \tag{1.14}$$

- The Einstein-Hilbert invariance action is

$$S = \int dx^4 \sqrt{-\det g}\, R. \tag{1.15}$$

1.2.2. Non-commutative geometry

Alain Connes proposed to extend Riemannian geometry and the concept of manifolds to non-commutative geometry (NCG) by replacing the algebra $C^{\infty}(\mathcal{M})$ with an arbitrary algebra [12, 13]. Thus, the three basic components to build NCG according to Connes are $(\mathcal{H}, D, A)$, with $\mathcal{H}$ is the Hilbert space of wave functions, D is the Dirac operator that satisfies the de Rham condition $D^2 = 0$, impact on the wave functions and A is any algebra that can be non-commutative and is an extension of algebra $C^{\infty}(\mathcal{M})$ [14].

CHAPTER 2. DISCRETIZED KALUZA-KLEIN THEORY-DKKT

2.1. DKKT with a extra dimension of two points.

2.1.1. Space-time with a discretized extra dimension

In 1995, Viet, Landi and Wali formulated the extended Hilbert-Einstein action for the NCG which obtained a theory that includes fields like the classical Kaluza-Klein theory including gravity, electromagnetic fields and a Brans-Dicke scalar field [15].

2.1.2. Algebra of smooth functions and generalized derivatives in DKKT

In DKKT, the customary algebra of smooth functions $C^{\infty}(M)$ is generalized to $A = C^{\infty}(M) \oplus C^{\infty}(M)$ and any generalized function $F \in A$, 0-forms, can be written as

$$F(x) = f_{+}(x)\mathbf{e} + f_{-}(x)\mathbf{r}, \tag{2.1}$$

with $e, r \in Z_2 = \{e, r \mid e^2 = e, r^2 = e, er = re = r\}$. We adopt a 2×2 matrix representation for $\mathbf{e}, \mathbf{r}$

$$e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, r = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.2}$$

Then the function $F(x)$ assumes a 2×2 matrix form and has a permutation ego function conjugated to it $\tilde{F}(x)$ as follows

$$F(x) = f_{+}(x)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + f_{-}(x)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} f_1(x) & 0 \\ 0 & f_2(x) \end{pmatrix}, \tilde{F}(x) = \begin{pmatrix} f_2(x) & 0 \\ 0 & f_1(x) \end{pmatrix}, \tag{2.3}$$

The action of the derivatives on the 0-form elements as given by

$$D_N(F) = [D_N, F], \tag{2.4}$$

satisfying the Newton-Leibnitz rule

$$D_N\left(FG\right) = D_N\left(F\ \right)G + F\ D_N\left(G\right). \tag{2.5}$$

2.1.3. General and orthonormal basis of 1-forms

In article [1] selected a diagonal representation for the curvi-linear basis DX^{μ} and $DX^{5}\sigma^{+}$ to construct generalized one- and higher forms in analogy with the usual Riemannian geometry. However, it is more convenient to work in a representation in which the vielbeins E^{A} are diagonal, with index $A = a, \dot{5} = 0,1,2,3,\dot{5}$ efers to the local flat basis system. Locally, E^{A} is given as follows

$$E^{a} = \begin{pmatrix} e^{a} & 0 \\ 0 & e^{a} \end{pmatrix},\ E^{\dot{5}} = \begin{pmatrix} 0 & \theta \\ \theta & 0 \end{pmatrix}, \tag{2.6}$$

In this basis the wedge product can be defined as follows

$$E^{A} \wedge E^{B} = -E^{B} \wedge E^{A}. \tag{2.7}$$

Both basis span the space of generalized 1-forms; hence an arbitrary 1-form U in NCG is given by

$$U = E^{A}U_{A} = DX^{M}U_{M}, \tag{2.8}$$

we can express them in terms of each other as follows

$$\begin{aligned} E^{A} &= DX^{M}E_{M}^{A}, \\ DX^{M} &= E^{A}E_{A}^{M}, \end{aligned} \tag{2.9}$$

Without any loss of generality we can choose E_{M}^{A} as follows

$$E_{\mu}^{a} = \begin{pmatrix} e_{1\mu}^{a}\left(x\right) & 0 \\ 0 & e_{2\mu}^{a}\left(x\right) \end{pmatrix} \quad , \quad E_{5}^{a} = 0$$

$$E_5^{\dot{5}} = \begin{pmatrix} \phi_1(x) & 0 \\ 0 & \phi_2(x) \end{pmatrix} = \Phi \quad , \quad E_\mu^{\dot{5}} = \begin{pmatrix} e_{1\mu}^{\dot{5}} & 0 \\ 0 & e_{2\mu}^{\dot{5}} \end{pmatrix} = -A_\mu \Phi. \qquad (2.10)$$

We obtain the components of the general derivative of form-1 as

$$\begin{aligned}(DU)_{bc} &= \frac{1}{2} E_b^\mu E_c^\nu \left(\partial_\mu E_\nu^a - \partial_\nu E_\mu^a\right) U_a - \frac{1}{2} E_b^\mu E_c^\nu F_{\mu\nu} \Phi U_{\dot{5}} \\ &+ \frac{1}{2}\left(E_b^\mu \partial_\mu U_c - E_c^\nu \partial_\nu U_b\right) + \frac{m}{2}\left[\begin{aligned}&\left(A_b.\tilde{E}_c^\nu - A_c.\tilde{E}_b^\nu\right) E_\nu^a U_a + \left(A_c \tilde{U}_b - A_b \tilde{U}_c\right) \\ &+ \left(A_b.\tilde{E}_c^\nu - A_c.\tilde{E}_b^\nu\right)\left(\tilde{A}_\nu - A_\nu\right) \Phi U_{\dot{5}}\end{aligned}\right],\end{aligned} \qquad (2.11)$$

$$\begin{aligned}(DU)_{b\dot{5}} &= \frac{1}{2} \tilde{E}_b^\mu \left(\frac{\partial_\mu \Phi}{\Phi} U_{\dot{5}} + \partial_\mu U_{\dot{5}}\right) + \frac{m}{2}\left[\Phi^{-1}\left(\tilde{U}_b - \tilde{E}_b^\mu E_\mu^c U_c\right)\right] \\ &+ \frac{m}{2}\left[\tilde{E}_b^\mu A_\mu U_{\dot{5}} - \tilde{A}_b\left(1 + \tilde{\Phi}^{-1}\Phi\right) U_{\dot{5}} + \tilde{A}_b \tilde{U}_{\dot{5}}\right].\end{aligned} \qquad (2.12)$$

2.1.4. Generalized metric

the components of the metric tensors G^{MN} and G_{MN} turn out to be

$$\begin{aligned} &G^{\mu\nu} = G^{\mu\nu} \doteq \begin{pmatrix} g_1^{\mu\nu} & 0 \\ 0 & g_2^{\mu\nu} \end{pmatrix}, && G_{\mu\nu} \doteq \begin{pmatrix} g_{1\mu\nu} & 0 \\ 0 & g_{2\mu\nu} \end{pmatrix} + A_\mu A_\nu, \\ &G^{\mu 5} = A^\mu = G^{5\mu}, && G_{\mu 5} = G_{5\mu} = A_\mu \Phi, \\ &G^{55} = \Phi^{-2} + A^2, && G_{55} = \Phi^2. \end{aligned} \qquad (2.13)$$

where $g_i^{\mu\nu} = e_{ia}^\mu \eta^{ab} e_{ib}^\nu, i = 1,2$ are the metric tensors on the two sheets.

2.2. The particle model of photon-fermion interactions

In this space-time, The discrete extra dimension having two points implies only one Kaluza-Klein partner for each fermions. An arbitrary fermion pair is represented by

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \tag{2.14}$$

and the Dirac operator $\not{\partial}$ is extended to

$$\not{D} = \Gamma^M D_M = \begin{bmatrix} \not{\partial} & -m\lambda^{-2} \\ m\lambda^{-2} & \not{\partial} \end{bmatrix}, \tag{2.15}$$

The photon 4 vector potential $\not{A}(x) = g\gamma^\mu A_\mu(x)$ is extended to the matrix

$$\not{B} = \begin{bmatrix} \gamma^\mu g A_\mu(x) & \sqrt{2}gH(x) - m\lambda^{-2} \\ \sqrt{2}gH(x) - m\lambda^{-2} & \gamma^\mu\left(gA_\mu(x)Q - g' X_\mu(x) Q_X\right) \end{bmatrix}, \tag{2.16}$$

where $A_\mu(x)$ is the usual electromagnetic 4- vector potential, the vector field $X_\mu(x)$ and the scalar $H(x)$ are KK partners of photon, Q and Q_X are, respectively, the electric and dark charge operators.

The Lagrangian for the extended vector field for photon is

$$L_g = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{g'^2}{4g^2}X^{\mu\nu}X_{\mu\nu} + g'^2\lambda^4 X_\mu^2(x)H^2(x) + L_H, \tag{2.17}$$

So in this model, $g = g'$, the H-photon has a vacuum expectation value $v_0 = m/\sqrt{2}g\lambda^2$ and a mass $m_H = \sqrt{2}m\lambda^{-2}$ due to the quartic potentia, giving a mass photon X is $m_X = m/\sqrt{2}$. Therefore, based on the experiment at ATOMKI, we define the mass of X as 17 MeV. Masses of X-photon and H-photon are contacted with mass parameter as follows

$$m_X = m/\sqrt{2} = 17MeV,\ m_H = 2m_X/\lambda^2,\ m = 24MeV \tag{2.18}$$

he total Lagrangian of all fermions generalizes the one $\bar{\psi}(x)(i\not{\partial} + m_0 \not{A}(x))\psi(x)$ as follows

$$L_{f-A} = \mathrm{Tr}\left[\bar{\Psi}(x)(i\not{D} + M_0 + \not{B}(x))\Psi(x)\right] = L_f + L_{int}, \qquad (2.19)$$

The mass matrices are respectively

$$M_0 = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \; M = \begin{bmatrix} m_1 & -m(1+i)\lambda^{-2} \\ -m(1-i)\lambda^{-2} & m_2 \end{bmatrix}. \qquad (2.20)$$

We can use the following unitary transformations to diagonalize the mass matrix and obtain the eigenstanal masses of the Kaluza-Klein partners of the fermions

$$U_f = \begin{bmatrix} \cos\theta_f & -e^{i\alpha_f}\sin\theta_f \\ e^{-i\alpha_f}\sin\theta_f & \cos\theta_f \end{bmatrix}. \qquad (2.21)$$

Choosing $\alpha_f = \pi/4$, we have

$$M_f = \begin{bmatrix} M_1^f + \left(M_2^f - M_1^f\right)\sin^2\theta_f & -\frac{1}{2\sqrt{2}}\left(M_2^f - M_1^f\right)\sin 2\theta_f\,(1+i) \\ -\frac{1}{2\sqrt{2}}\left(M_2^f - M_1^f\right)\sin 2\theta_f\,(1-i) & M_1^f + \left(M_2^f - M_1^f\right)\cos^2\theta_f \end{bmatrix}. \qquad (2.22)$$

Compare with (2.20), we have the mass splitting formula between a regular fermion particle ψ and its KK partner ψ_X according to the mixing angle is

$$M_2^f - M_1^f = m_f - m_{f_X} = \frac{2\sqrt{2}m}{\sin 2\theta_f \lambda^2} = \frac{4m_X}{\sin 2\theta_f \lambda^2}. \qquad (2.23)$$

The interacting Lagrangian L_{int} in terms of the fermion's mass eigenstates is

$$L_{f-A} = g\left(\bar{\psi}(x)\not{A}(x)\psi(x) + \bar{\psi}_X(x)\not{A}(x)\psi_X(x)\right),$$

$$L_{f-X} = g\Big(\sin^2\theta_f \bar{\psi} \not{X}(x)\psi + \cos^2\theta_f \bar{\psi}_X \not{X}(x)\psi_X$$

$$+\frac{1}{2}\sin 2\theta_f \bar{\psi}_X \not{X}(x)\psi + \frac{1}{2}\sin 2\theta_f \bar{\psi} \not{X}(x)\psi_X\Big), \quad (2.24)$$

$$L_{f-H} = g\sin 2\theta_f H(x)(-\bar{\psi}\psi + \bar{\psi}_X\psi_X)$$
$$+ g\cos^2\theta_f H(x)(-\bar{\psi}\psi_X + \bar{\psi}_X\psi).$$

Hence, we can easily obtain the interaction constants between dark photon H and dark photon X with fermions. In this thesis, I will consider the problem of neutron decay, therefore, we can have an explicit coupling of neutrino, neutron and their KK partners with the K-K partners of photon as follows [16]

$$\begin{aligned}
&g_{Xnn} = g\sin^2\theta_n,\ g_{Xn_Xn_X} = g\cos^2\theta_n,\ g_{Xnn_X} = 1/2g\sin 2\theta_n,\\
&g_{X\nu\nu} = g\sin^2\theta_\nu,\ g_{X\nu_X\nu_X} = g\cos^2\theta_\nu,\ g_{X\nu\nu_X} = 1/2g\sin 2\theta_\nu,\\
&g_{Hnn} = g_{Hn_Xn_X} = g\sin^2\theta_n,\ g_{Hnn_X} = g\cos^2\theta_n,\\
&g_{H\nu\nu} = g_{X\nu_X\nu_X} = g\sin^2\theta_\nu,\ g_{H\nu\nu_X} = g\cos^2\theta_\nu,
\end{aligned} \quad (2.25)$$

with θ_n, θ_ν are respectively the mixing angles of neutron and neutrino with their Kaluza-Klein partners.

Based on the new elementary particle model and the exclusion of the electron and anti-electron pair decay channel, consistent with the experimental results of Tang et al. [17], we make the following additional assumptions

$$m_n - m_{n_X} < 1{,}102\,\text{MeV}. \quad (2.26)$$

CHAPTER 3. NON-ABELIAN GAUGE FIELDS AS COMPONENTS OF GRAVITY IN DKKT

3.1. Connection, torsion and curvature

3.1.1. Hermitian and metric compatible connection 1-forms

Levi-Civita connection 1-form Ω_{AB} satisfies

$$\Omega^{+}_{AB} = -\Omega_{BA}, \quad \Omega^{+}_{AB} = \Omega_{AB}. \tag{3.1}$$

3.1.2. The first structure equation and torsion 2-forms

The first Cartan structure equation defines the torsion 2-forms T^A as given by

$$T_A = DE_A - E^B \wedge \Omega_{AB}. \tag{3.2}$$

In this, we shall assume

$$T_{abc} = T_{ab\dot{5}} = T_{a\dot{5}b} = 0, \quad T_{\dot{5}AB} = t_{\dot{5}AB} r. \tag{3.3}$$

The components of the connection as

$$\Omega_{abc} = \frac{1}{2}\begin{bmatrix} E_b^{\mu} E_c^{\nu}\left(\partial_{\mu} E_{a\nu} - \partial_{\nu} E_{a\mu}\right) + E_c^{\mu} E_a^{\nu}\left(\partial_{\mu} E_{b\nu} - \partial_{\nu} E_{b\mu}\right) \\ -E_a^{\mu} E_b^{\nu}\left(\partial_{\mu} E_{c\nu} - \partial_{\nu} E_{c\mu}\right) \end{bmatrix}$$
$$+\frac{m}{2}\begin{bmatrix} \left(A_b.\tilde{E}_c^{\nu} - A_c.\tilde{E}_b^{\nu}\right)E_{\nu}^{a} + \left(A_c.\tilde{E}_a^{\nu} - A_a.\tilde{E}_c^{\nu}\right)E_{\nu}^{b} \\ -\left(A_a.\tilde{E}_b^{\nu} - A_b.\tilde{E}_a^{\nu}\right)E_{\nu}^{c} + 2\left(A_a\eta_{bc} - A_b\eta_{ac}\right) \end{bmatrix}.$$

$$\Omega_{ab\dot{5}} = \frac{1}{4}\left(E_a^{\mu} E_b^{\nu} F_{\mu\nu}\Phi + \tilde{E}_a^{\mu}\tilde{E}_b^{\nu}\tilde{F}_{\mu\nu}\tilde{\Phi}\right) + \frac{m}{4}\begin{bmatrix} \Phi^{-1}\left(\tilde{E}_a^{\mu} E_{b\mu} - \tilde{E}_b^{\mu} E_{a\mu}\right) \\ +\tilde{\Phi}^{-1}\left(E_a^{\mu}\tilde{E}_{b\mu} - E_b^{\mu}\tilde{E}_{a\mu}\right) \end{bmatrix}$$
$$+\frac{m}{4}\left[\left(\tilde{A}_a.E_b^{\nu} - \tilde{A}_b.E_a^{\nu}\right)\tilde{\Phi} - \left(A_a.\tilde{E}_b^{\nu} - A_b.\tilde{E}_a^{\nu}\right)\Phi\right]\left(\tilde{A}_{\nu} - A_{\nu}\right).$$

$$\Omega_{\dot{5}ab} = -\frac{1}{4}\left(E_a^\mu E_b^\nu F_{\mu\nu}\Phi + \tilde{E}_a^\mu \tilde{E}_b^\nu \tilde{F}_{\mu\nu}\tilde{\Phi}\right) + \frac{m}{4}\Big[\Phi^{-1}\left(4\eta_{ab} - 3\tilde{E}_b^\mu E_{a\mu} - \tilde{E}_a^\mu E_{b\mu}\right)$$
$$+\tilde{\Phi}^{-1}\left(E_b^\mu \tilde{E}_{a\mu} - E_a^\mu \tilde{E}_{b\mu}\right) - \left[\left(\tilde{A}_a.E_b^\nu - \tilde{A}_b.E_a^\nu\right)\tilde{\Phi} - \left(A_a.\tilde{E}_b^\nu - A_b.\tilde{E}_a^\nu\right)\Phi\right]\left(\tilde{A}_\nu - A_\nu\right)\Big].$$

$$\Omega_{\dot{5}b\dot{5}} = \left(\tilde{D}E_{\dot{5}}\right)_{b\dot{5}} - \left(DE_{\dot{5}}\right)_{b\dot{5}} = \frac{1}{2}\left(\tilde{E}_b^\mu \frac{\partial_\mu \Phi}{\Phi} + E_b^\mu \frac{\partial_\mu \tilde{\Phi}}{\tilde{\Phi}}\right)$$
$$+\frac{m}{2}\left[\tilde{E}_b^\mu\left(A_\mu - \tilde{A}_\mu \tilde{\Phi}^{-1}\Phi\right) + E_b^\mu\left(\tilde{A}_\mu - A_\mu \Phi^{-1}\tilde{\Phi}\right)\right].$$

$$\Omega_{\dot{5}\dot{5}a} = \Omega_{\dot{5}\dot{5}\dot{5}} = 0. \tag{3.4}$$

3.1.3. Second structure equation and Ricci curvature tensor

The second Cartan structure equation defines curvature 2-forms as follows

$$R_{AB} = D\Omega_{AB} + \Omega_{AC} \wedge \Omega^C{}_B. \tag{3.5}$$

We have the expression for the Ricci curvature tensor in the Abel case as

$$R = R_4 + \frac{m^2}{16}\left(-152 + 16E^{a\mu}\tilde{E}_{a\mu} + 64\tilde{E}^{a\mu}E_{a\mu} - G_{\mu\nu}\tilde{G}^{\mu\nu} - \tilde{G}_{\mu\nu}G^{\mu\nu} - 8\left(\tilde{E}^{a\mu}E_{a\mu}\right)^2\right)$$
$$+\frac{m^2}{16}\left[-7E_{a\nu}\tilde{E}^{a\mu}E_{b\mu}\tilde{E}^{b\nu} + E^{a\mu}\tilde{E}_{a\nu}E^{b\nu}\tilde{E}_{b\mu} - 2\tilde{E}^{a\mu}E_a^\nu \tilde{E}_{b\nu}E_\mu^b\right]. \tag{3.6}$$

3.2. Non-Abelian gauge fields as components of gravity

For simplicity when considering the non-Abelian case, we choose

$$E_\mu^a = \begin{pmatrix} e_\mu^a(x) & 0 \\ 0 & e_\mu^a(x) \end{pmatrix} = e_\mu^a(x)\mathbf{e}, \qquad E_5^a = 0,$$
$$E_5^{\dot{5}} = \begin{pmatrix} \phi & 0 \\ 0 & \phi \end{pmatrix} = \Phi, \qquad E_\mu^{\dot{5}} = -\begin{pmatrix} a_{\mu L}\phi & 0 \\ 0 & a_{\mu R}\phi \end{pmatrix} = -A_\mu \Phi. \tag{3.7}$$

We obtain the final formula of the Ricci curvature tensor as

$$R=\frac{1}{2}\begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}-g^{\mu\nu}\frac{\partial_\mu\partial_\nu\phi}{\phi}-g^{\alpha\nu}e^{a\mu}\partial_\alpha e_{a\mu}\frac{\partial_\nu\phi}{\phi}$$
$$-\frac{1}{8}\phi^2 g^{\mu\alpha}g^{\nu\beta}\left(f_{+\mu\nu}+2m\left[a_{-\mu},a_{-\nu}\right]\right)\left(f_{+\alpha\beta}+2m\left[a_{-\alpha},a_{-\beta}\right]\right). \quad (3.8)$$

The third term in the expression does not contribute to Lagrangian so we ignore it. The curvature tensor can ignore a coefficient, so we obtain the following shortened form

$$R=R_4-\frac{2\Box\phi}{\phi}-\frac{1}{4}\phi^2 g^{\mu\alpha}g^{\nu\beta}\hat{f}_{\mu\nu}\hat{f}_{\alpha\beta}, \quad (3.9)$$

where the first term is the known 4-dimensional curvature, and the last term contains the non-Abelian component

$$\hat{f}_{\mu\nu}(x)=f_{+\mu\nu}+2m\left[a_{-\mu},a_{-\nu}\right]$$
$$=\partial_\mu a_{+\nu}(x)-\partial_\nu a_{+\mu}(x)+2m\left[a_{-\mu}(x),a_{-\nu}(x)\right]. \quad (3.10)$$

Write as left and right components on two sheets

$$\hat{f}_{\mu\nu}(x)=\frac{1}{2}\left\{g_{\mu\nu L}+g_{\mu\nu R}-\frac{m}{2}\left(\left[a_{\mu L},a_{\nu R}\right]+\left[a_{\mu R},a_{\nu L}\right]\right)\right\}. \quad (3.11)$$

where the covariant field strengths are defined as

$$g_{\mu\nu L,R}=\partial_\mu a_{\nu L,R}-\partial_\nu a_{\mu L,R}+m\left[a_{\mu L,R},a_{\nu L,R}\right]. \quad (3.12)$$

The Ricci scalar curvature in Eq. (3.9) is gauge invariant only if the non-covariant terms in Eq. (3.11) vanish or contribute to the covariant field strength. So, we have the following two cases:

a) Case 1: One gauge field is Abelian.

We can introduce the physical Abelian vector fields $B_\mu(x)$ and possibly non-Abelian $W_\mu(x)$ ones as follows

$$a_{\mu R} = \frac{g'}{m} B_\mu(x) Y_R, \qquad a_{\mu L} = \frac{g'}{m} B_\mu(x) Y_L - \frac{g}{m} W_\mu(x). \tag{3.13}$$

where $B_\mu(x)$ is an Abelian gauge vector and $W_\mu(x) = W_\mu^i(x) T^i$ are non-Abelian gauge vectors , m are mass parameters, Y_L, Y_R are the sum of generators of the left- and right-handed Abelian gauge groups

$$\hat{f}_{\mu\nu} = \frac{g'}{2m}(Y_L + Y_R) B_{\mu\nu} + \frac{g}{2m} \mathrm{W}_{\mu\nu}, \tag{3.14}$$

We have the Hilbert–Einstein action

$$S = \frac{1}{16\pi G_N} \int dx^4 \sqrt{-\det|g|}\,\phi(x) \frac{1}{2N} \mathrm{Tr}(R). \tag{3.15}$$

Where G_N is Newton's constant, substitute the above formulas (3.15) we have

$$\begin{aligned} S_g &= -\frac{1}{64N\pi G_N} \int dx^4 \sqrt{-\det g}\,\phi^3 g^{\mu\rho} g^{\nu\tau} \mathrm{Tr}\left(\hat{f}_{\mu\nu} \hat{f}_{\rho\tau}\right) \\ &= \frac{1}{4} \int dx^4 \sqrt{-\det g}\,\phi^3 \left(\frac{g'^2}{m^2} \frac{\sum (Y_L + Y_R)^2}{64N\pi G_N} B^{\mu\nu} B_{\mu\nu} + \frac{g^2}{64N\pi G_N m^2} \mathrm{Tr} W^{\mu\nu} W_{\mu\nu} \right). \end{aligned} \tag{3.16}$$

In order to keep the correct kinetic terms for the gauge fields B_μ and W_μ, we must have two model-dependent relations between the parameters g', g, m and G_N. In SM, we have $N = 8$, the Abelian gauge group $U(1)$ and the non-Abelian one $SU(2) \times U(1)$. The relations between the parameters are

$$g = 16m\sqrt{2\pi G_N}, \quad g' \sum (Y_L + Y_R)^2 = 16m\sqrt{\pi G_N}. \tag{3.17}$$

In any case, the generalized Einstein–Hilbert action is reduced to a gauge-invariant physical theory if the gauge fields on one sheet of spacetime are Abelian.

b) Case 2: Two non-Abelian gauge fields must have the same form

Let us introduce the physical non-Abelian gauge field $C_\mu(x) = C_\mu^i(x)\lambda^i$ specific to the color group

$$a_{\mu L} = \frac{\alpha g_S}{m} C_\mu, \quad a_{\mu R} = \frac{g_S}{m} C_\mu, \quad a_{\pm\mu} = (\alpha \pm 1)\frac{g_S}{2m} C_\mu. \tag{3.18}$$

The field strength now has the covariant form

$$\begin{aligned} \hat{f}_{\mu\nu} &= (\alpha+1)\frac{g_S}{2m} C_{\mu\nu}, \\ C_{\mu\nu} &= \partial_\mu C_\nu - \partial_\nu C_\mu + g_S \left[C_\mu, C_\nu\right]. \end{aligned} \tag{3.19}$$

Is $\alpha = 3$, the kinetic term of the vector field C_μ will have the correct factor if we choose

$$g_S = 2\sqrt{\pi G_N}\, m. \tag{3.20}$$

Then, the Hilbert–Einstein action is a gauge-invariant

$$S_g = -\frac{1}{2}\int dx^4 \sqrt{-\det g}\,\phi^3 C^{\mu\nu} C_{\mu\nu}. \tag{3.21}$$

If we choose the color SU (3) là một nhóm gauge phi Abel, as the non-Abelian gauge group, then the action (3.21) becomes the action of QCD coupled to gravity.

CHAPTER 4. DECAY WIDTH OF NEUTRON IN THE NEW DECAY CHANNELS OF THE DKKT

4.1. Lifetime puzzle and new dark decay channels of neutron in DKKT

According to SM, the neutron lifetime with β channel is $\tau_{SM} = 878,7 \pm 0,6$ [18]. This value is perfectly compatible with the neutron lifetime measured in the "bottle" measurement, $\tau_{bottle} = 879,4 \pm 0,6$ [19]. However, the lifetime measured in the beam experiments via counting the produced protons gets 8 seconds longer, about 1% with $\tau_{beam} = 888 \pm 0,2$ [20].

In DKKT with the extended particle model, we assume that there are totally 6 possible neutron decay channels included

$$i)\quad n \xrightarrow{W} p + e^- + \bar{\nu}_e\,, \qquad iv)\quad n \to n_X + \nu + \bar{\nu}_X\,,$$
$$ii)\quad n \to n_X + H, \qquad v)\quad n \to n_X + \nu_X + \bar{\nu},$$
$$iii)\quad n \xrightarrow{X17} n_X + \nu + \bar{\nu}, \qquad vi)\quad n \to n_X + \nu_X + \bar{\nu}_X\,.$$

4.2. Neutron decay with H photon production

We have Feynman diagram

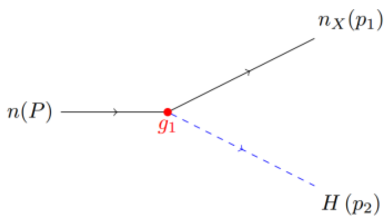

Figure 1. *Decay of neutron to the dark one and the H photon.*

The decay rate of neutron with the H-photon production channel

$$\Gamma_H = \frac{2g^2 \cos^4 \theta_n m_X}{\pi \lambda^2} \left(\frac{4}{\sin^2 2\theta_n} - 1 \right)^{1/2} \sim \frac{2g^2 m_X \cos^3 \theta_n}{\pi \lambda^2 \sin \theta_n}. \quad (4.1)$$

4.3. The dark decay of neutron via X17

We see that all neutron decay channels through interacting particle X17 (iii, iv, v and vi) are described by the following diagram.

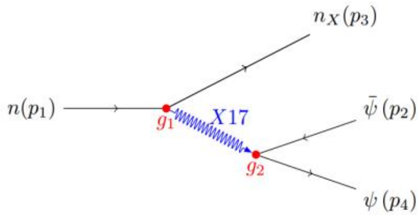

Figure 2. *Decays of neutron via X17 to the dark one and a pair of leptons.*

Applying the rules for calculating Feynmann diagrams in the field theory, we obtain the differential expression of the decay width according to the energy of the ejected neutrinos

$$d\Gamma = \frac{\langle |M|^2 \rangle}{(4\pi)^3 \hbar m_1} dE_2 dE_4 . \quad (4.2)$$

To calculate the decay width, we add the following dimensionless parameters of the same order

$$\varepsilon = \frac{m_n - m_{n_X}}{m_n}, \ \eta = \frac{E_4}{m_1} = \frac{E_4}{m_n}, \gamma = \frac{m_2}{m_1} = \frac{m_{\bar{f}}}{m_n}, \delta = \frac{m_4}{m_1} = \frac{m_f}{m_n},$$
$$\phi = \sqrt{\eta^2 - \delta^2} = \frac{|\vec{p}_4|}{m_n}, E = \frac{E_2}{m_1} = \frac{E_2}{m_n}, \ X = \frac{M_X^2}{m_1^2} = \left(\frac{M_X}{m_n} \right)^2 . \quad (4.3)$$

We obtain the expression for the two bounds of E as

$$E_{\pm} \approx \varepsilon - \eta + \frac{\gamma^2 + \delta^2 - \varepsilon^2}{2} + \eta\varepsilon - \eta^2 \pm \phi\sqrt{(\varepsilon - \eta)^2 - \gamma^2} \ . \tag{4.4}$$

Conditions of E_4 is $\Delta' > 0$ corresponds to the conditions of η is

$$\delta < \eta < \varepsilon - \gamma \ . \tag{4.5}$$

We have obtained the general expression of the differential scattering amplitude according to the energy of the 4th particle.

$$\begin{aligned} \frac{d\Gamma}{d\eta}\left(\frac{g_1^2 g_2^2 m_n}{4\pi^3\hbar X^2}\right)^{-1} &= -4\phi^2\left[(\varepsilon-\eta)^2 - \gamma^2\right] \\ &-\phi\sqrt{(\varepsilon-\eta)^2-\gamma^2}\left(8\eta^2 - 8\eta\varepsilon - 2\delta^2 - 2\gamma^2 - 12\gamma\delta\right). \end{aligned} \tag{4.6}$$

We will calculate this integral in each case of decay via X17.

4.3.1. Neutron decay into neutrino and anti-neutrino pairs

$$\Gamma_3 = \frac{g_1^2 g_2^2 m_n}{30\pi^3\hbar X^2}\varepsilon^5 = \frac{g_1^2 g_2^2}{30\pi^3\hbar M_X^4}\left(m_n - m_{n_X}\right)^5 . \tag{4.7}$$

4.3.2. Neutron decay into dark neutrino and anti-neutrino pairs

$$\begin{aligned} \Gamma_4 &= \frac{g_1^2 g_2^2}{30\pi^3\hbar M_X^4}\left(m_n - m_{n_X}\right)^5 \left[\begin{aligned} &-1 + 10a^2 - 20a^3 + 15a^4 - 4a^5 - \frac{15}{2}a^4\ln a \\ &+\frac{\sqrt{1-a^2}}{2}\left(4 - 13a^2 - 6a^4\right) + \frac{15}{2}a^4\ln\left(1+\sqrt{1-a^2}\right) \end{aligned}\right] \\ &= \frac{g_1^2 g_2^2}{30\pi^3\hbar M_X{}^4}\left(m_n - m_{n_X}\right)^5 J_4(a). \end{aligned} \tag{4.8}$$

Where $a = \frac{\delta}{\varepsilon} = \frac{m}{m_n - m_{n_X}} = \frac{m_{\nu_X}}{m_n - m_{n_X}}$.

Use Mathematica software with conditions $0 \le a < 1$ we have the maximum value of J_4 is approximately 1,009 when $a \approx 0{,}11186$.

4.3.3. Neutron decay into neutrino and dark anti-neutrino pairs

$$\Gamma_5 = \Gamma_4. \tag{4.9}$$

4.3.4. Neutron decay into dark neutrino and dark anti-neutrino pairs

We obtain the final expression of the channel decay width (vi) as:

$$\Gamma_6 = \Gamma_3 . J_6(a) \approx \frac{g_1^2 g_2^2}{30\pi^3 \hbar M_X^4} \left(m_n - m_{n_X} \right)^5 \begin{pmatrix} 1 + 21{,}435 a^2 - 65{,}94 a^3 \\ -\ 74{,}94 a^4 + 209{,}235 a^5 \end{pmatrix} \tag{4.10}$$

Maximum value of J_6 is approximately 1,28 when $a \approx 0{,}198477$.

4.4. Comparison of new neutron decay channels

We have, the total decay width of the decay channels via X17 is

$$\Gamma_X = \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 < 1{,}28 \frac{16 g^4 m_X}{15\pi^3 \lambda^{10} \sin^3\theta_n \cos^3\theta_n} . \tag{4.11}$$

The ratio of the decay width between two the cases, decay to scalar particle H and decay via X17 is

$$\frac{\Gamma_X}{\Gamma_H} < \frac{256 g^2}{15\pi^2 \sin^4 2\theta_n \lambda^8} < \frac{256 g^2}{15\pi^2\ 61{,}7^4} < 1{,}2\,10^{-7} g^2 << 1 . \tag{4.12}$$

Thus, we see that if the scalar decay channel (ii) exists, which means the mass condition $m_H < 1{,}102\text{MeV}$ is satisfied, the decay channel (ii) plays the main role in the dark decay channels and we can ignore the decay channels via X17.

4.5. Vấn đề phân rã neutron và lời giải trong lý thuyết DKKT

Thus, in the new particle model of DKKT, have new decay channels of neutron. For simplicity, we ignore the decay channels through X17, and only consider the β decay channel and the H decay channel. The lifetime or half-life measured in the bottle experiment is the total half-life of the neutron, and the lifetime in the beam experiment is the time for the number of protons produced to equal half the number of initial neutrons. So we have the relationship between the half-lives of channels (i) and (ii) as

$$\frac{\lambda_1}{\lambda_1+\lambda_2}\left(1-e^{-\ln 2\frac{\tau_{beam}}{\tau_{bottle}}}\right)=\frac{1}{2}. \tag{4.13}$$

Thus, we obtain the ratio of the decay constant or decay width according to the two channels as

$$\frac{\Gamma_H}{\Gamma_{\mathrm{w}}}=\frac{\Gamma_{ii}}{\Gamma_i}=\frac{\lambda_2}{\lambda_1}=1-2e^{-\ln 2\frac{\tau_{beam}}{\tau_{bottle}}}=1-2^{1-\frac{\tau_{beam}}{\tau_{bottle}}}\sim 6,756.10^{-3}. \tag{4.14}$$

We obtain the equation relating the parameters of the DKKT theory based on the two beam and bottle experiments as

$$\frac{\cos^3\theta_n}{\lambda^2\sin\theta_n}=\left(1-2^{1-\frac{\tau_{beam}}{\tau_{bottle}}}\right)\frac{4,752g^2m_e^5}{64\pi^2m_X m_W^4\sin^4\theta_W}\sim\frac{g^2m_e^5 10^{-3}}{2\pi^2m_X m_W^4\sin^4\theta_W}. \tag{4.15}$$

Thus, the problem of the difference in neutron lifetime when measured by the beam and bottle methods has been solved in DKKT.

CONCLUSIONS

- We have successfully incorporated non-Abelian gauge fields into the veilbien, which appear as components of the gravitational field when existing in the Ricci curvature tensor. This is the mathematical basis for DKKT theory to unify interactions in a geometric theory.
- The result is the very strong property that the Hilbert-Einstein effect is only gauge invariant when the gauge field on the two leaves of the left-hand particle and the right-hand particle satisfy one of two cases:
  - The gauge field must be Abelian on a leaf, this is consistent with the properties of the weak interaction.
  - The non-Abelian gauge field must have the same shape on both leaves, consistent with strong interactions.
- We have proposed new neutron decay channels based on the Kaluza-Klein companions of elementary particles in the DKKT and experimental evidence.
- Develop procedures and techniques for calculating the decay width of new neutron decay channels.
- The decay width of the decay channels through X17 is very small compared to the scalar decay channel H, so we can ignore it if the decay channel H exists.
- Completely solve the problem of neutron lifetime in DKKT by using data from two the experiments, “beam” and “bottle”, to obtain the relationship between the parameters of the model.

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LIST OF THE PUBLICATIONS RELATED TO THE DISSERTATION

1. Nguyen Ai Viet, **Pham Tien Du,** 2017, *Non-abelian gauge fields as components of gravity in the discretized Kaluza-Klein theory*, Modern Physics Letters A, Vol. 32, No. 18 (2017) 1750095.
2. **Pham Tien Du**, Nguyen Ai Viet, Nguyen Van Dat, 2020, *Decay of neutron with participation of the light vector boson X17*, J. Phys.: Conf. Ser. 1506 012004
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